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Research article

Applying Simulink to solve the differential equation of rotational angle of a vibrating building resting on a viscous foundation

Tham Hong Duong[*]

ABSTRACT

Dynamic effects on structures are complicated topics, both in theoretical and experimental aspects. This article aims to study more deeply the effects of the waves on the response of a receiver footing in the vicinity. This paper establishes the governing equation for the vibrating structure resting on a viscous soil foundation. By prescribing all the rotating motions happened to the footings, including the sliding moments, and the anti-sliding moments, a differential equation is established. The viscous characteristics that based on a laboratory test-based results is considered, and the dynamic equilibrium equation of all the moments is manipulated to an ordinary diffferential equation. Beside the analytical solution, solved by the conventional approach, a Matlab Simulink diagram is created to solve the equation. The result indicates there is a value of rotational angle at which the rocking structure would attain during the process of vibration in long enough time. This solution could explain the failure mode of buildings which are constructed nearby a road, are stirred and vibrated under the dynamic effects caused by traffic mobility. These dynamic effects are of low-frequency range, or between 0 to 100 Hz.

A model of the finite element method, Plaxis 2D is used to validate the vibrational response of the receiver footing, concerning the effects on the far field. The soil foundation is very compressible, subjected to a vibration at different excitation frequencies. The movement of particles on the free domain of the soil surface at a distance of 50 m apart from the source of excitation results in a differential settlement for points within a receiver footing. And the displacement responses of the structure are analysed both in time domain and frequency domain. The beneficial application of Matlab Simulink for solving specific problems in Engineering Mechanics is suggested.

Key words: Displacement Response, Governing equation, Matlab Simulink tool, Viscous foundation

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INTRODUCTION

During the construction in urban regions, the vibrations caused by the operating machinery often spread to the buildings in the vicinity, and cause damage to the surrounding structures (Figure 1). Vibration emitted by construction activitie travels to the building around and stirs them vibrating. For the superstructure parts of these buildings, the response vibrations can be analysed by applying the simple models of structural dynamics; but for the subground parts of the building, the foundation that rests onto the soil foundation, it is a much more complicated problem; it still needs a broad knowledge of interdisciplinary aspects including mathematics, structural mechanics and soil dynamics, and sometime even skill of modeling, laboratory test or site test as well.

This study presents some preliminary results of analysis on the response of such a structure overlaying a foundation, participating in vibration caused by the external dynamic effects.

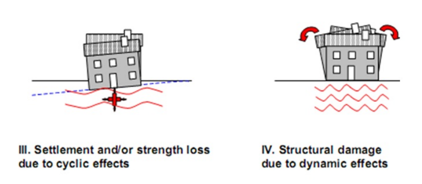

Figure 1: Vibration propagating through soil medium can alter the stress-strain relationship and cause the building quaking[1].

THEORETICAL BACKGROUND

Dynamic effects cause the waves propagating through the soil; during the wave travels, an arbitrary particle in the free domain displaces retrogressively. Under the self-weight of the receiver structure, the additional inertia forces, both in the vertical and horizontal direction, cause the receiver structure displaced more and damages occur from this displacement. There is

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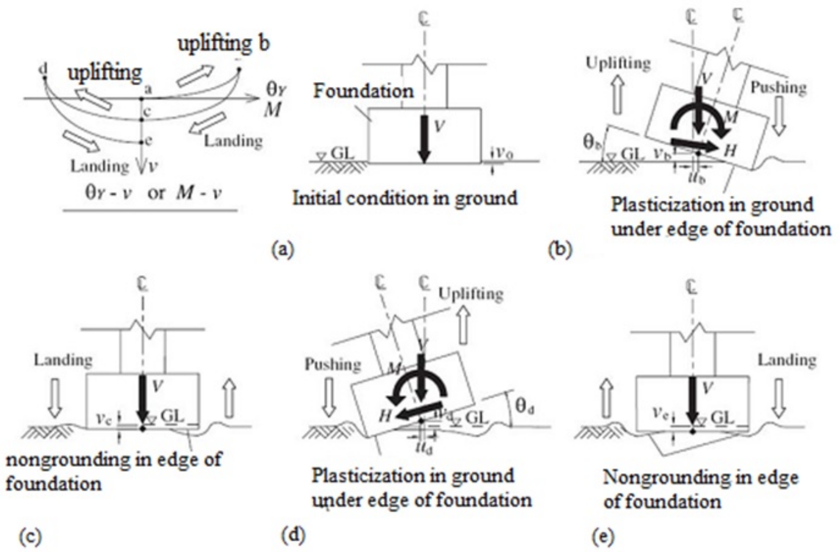


Figure 2: Uplifting and rolling sidesway reduce the area of base contact due to vibration [2].

evidence explaining the additional settlement of a receiver structure due to dynamic effects. In the basic concept of soil mechanics, settlement is the sum of accumulated vertical deformations within a definite region in the foundation. However, the increase of pore water pressure results in an additional settlement, occurring during the dissipation of the pressure with time, leading to the increase of the effective stress in some depth of the soil foundation. The displacement response of the foundation, when subjected to vibration, increases the additional forces and moments in terms of inertial forces and moments. The heavier the weight according to the type of building, the greater the inertia force, and the effective pressure on the foundation will be increased in a reducing area of contact (see Figure 2).

Literature reviews

Dynamic effects are vibrational action originating from a source and propagating through the medium to attack a receiver objective. A typical kind of dynamic effect is pile driving which is popular in construction activities for buildings. In some specific cases, pile driving is cheap and efficient for increasing the bearing capacity for the foundation, so this procedure is still applicable. Piles are cast in factories, installed into the soil by dropping the hammer with a driving frequency of a few cycles per second. As such the frequency is rather low as compared to that of machinery vibration which operates at thousands of cycles per second. By applying an impact load exerted on the pile head, this driving force causes an axial wave propagating along the structure shaft; if the weight of the falling part is heavier than that of the pile, it makes the pile move downward, together with the displaced soil around the pile. Vibration running through the shaft can move along the longitudinal axis of the structure and bounding from the pile tip to the head, causing tensile stress in localities around the pile head; this stress is calculated to the square root of the falling height of the hammer [3,4]. The maximum stress at the top of a pile is tensive stress to some extent causes cracks in reinforced concrete pile subjected to impact driving force. The energy provided from the hammer is available at some percent at hammer/pile ratio and its loss is due to partly delivering to cushion, friction effects...etc. Besides, waves are radiating from the pile tip to the surrounding soil medium, running spherically, and propagating up to the ground surface, coming to stir some receiver foundations of the buildings in the vicinity. This is a typical kind of dynamic effect which in some specific cases, intense vibration can cause serious damages to these facilities [4]. A lot of studies were conducted to determine the responsive vibration of the soil medium during the process of a wave traveling and the change in physical-mechanical properties under vibration (Dowding, 1996) [3], (Gert Degrande, 2002) [5], etc. Many prior research works studied intensively the dynamic effects of pile driving on the properties of soil medium. From the dynamic analysis of pile driving, the drivability of a solid prismatic pile is assessed by a numerical model in which the inertia effects of the soil around a pile, and the viscous nature of the soil is taken into account (Smith and Chow, 1982)[6]; a series of curves expressing the relationship between the penetrating rate (also called blow count) and the assumed static resistance of the pile[4]. In soil mass subjected to vibration, the excess pore water pressure also increases to several times of the cohesion especially at the surrounding surface of the pile as a result of the decrease in effective stress, and the coefficient of horizontal consolidation and the properties of soil compressibility are changed simultaneously (Randolph et al., 1979)[7], (Dowding, 1996)[3]. For mitigating the negative impact of the vibration due to pile driving, several measures of screening the vibration are suggested[6,8]. A footing subjected to a combined of vertical loads (i.e., static and inertia components) and moment could always rotate a specific time-dependent angle θ(t).

As such, it can be seen that: the displacement of the foundation in the near field of the receiver structure will be a result of the rotation angle θ(t) of the sliding footing.

Mathematical model

For establishing the mathematical model that governs the phenomena of a structure vibrating, all the loads and moments are taken into account. For a block of footing resting on a cohesive soil (i.e., viscous foundation), an assumption is that the sliding mass will rotate around a center laying on an axis on the edge of the foundation[9] (see Figure 3). The rotational displacement response of the foundation on cohesive soil is the solution of the differential equation that governs the motion of the vibrating foundation, in which the soil shear strength decreases with time.

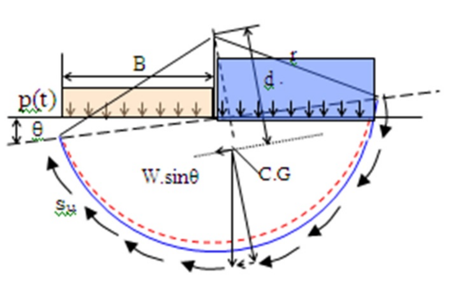


Figure 3: All the load exerting onto the footing with added mass rotating around a center laying on the edge of the structure[10].

The factor of safety against sliding is the ratio between the resisting moments divided by sliding moment, as below[11]:

$$\eta = \frac{M_I + M_G + M_{Su} + M_{dap} + M_{CF}}{M_{pq}} = \frac{M_{resist}}{M_{driving}} \tag{1}$$

where M$_I$ = moment due to rotating inertia rotating mass, MI = $J.\ddot{\theta} = mr^2\ddot{\theta}$; MG = moment of sliding mass $M_G = \pm\Sigma Wd.(\sin\theta \sim \theta)$, in which W is the weight of sliding mass; M$_S$ resisting moment due to mobilized soil strength, s$_{mob}$, $M_{Su} = \alpha R \Sigma s_{mod} \triangle s_i = \kappa\pi B.s_{mod}$; M$_{dap}$ = moment due to the soil mass overlying the level of footing. $M_{dap} = \frac{1}{2}\left[q_o + q(t)\right](r.\cos\theta \to B)^2$;

and M$_{CF}$ = moment due to viscous resistance, in terms of velocity of vibrational response, or $M_{CF} = \mu_{dyn}\left(N.\dot{\theta}\right)$.

About the sliding moment, M$_{pq}$ due to vertical load p(t) and q$_b$, M$_{pq}$ = 1/2(q$_b$+ p(t))B^2.

As for the viscous material characterized by the time-dependent soil strength, the law of attenuation is:

$$S_u(t) = \psi(t)s_o = m_1 exp(-nt)s_o \tag{2}$$

in which, m$_1$ being the constant concerning the effects of normal stress on the mobilized shear strength of sample tested, determined by laboratory test; n is a factor, taking the kind of soil and the frequency of excitation into account[10]. The mobilized soil strength is experimentally obtained as in Figure 4.

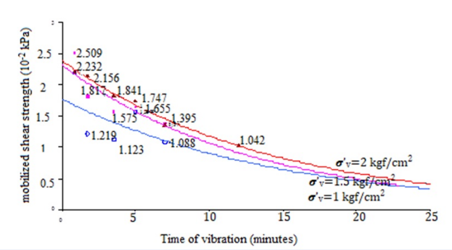


Figure 4: Quantifying the mobilized shear strength by lab tests[10].

The expression of the factor of safety could be rewritten as below:

$$\eta = \frac{\frac{W}{g}(\bar{r})^2\ddot{\theta} + Wd.\sin\theta + \left(\kappa B s_{mod} + k\eta_{eq}\dot{\theta}\right)}{\frac{1}{2}\left[q_b + p(t)\right]\times B^2} + \frac{\frac{1}{2}\left[q_b + p(t)\right]\times B^2 + \mu_{dyn}N}{\frac{1}{2}\left[q_b + p(t)\right]\times B^2} = 1 \tag{3}$$

In pseudo-static approach, the equilibrium condition is that η equals unity. This results in

$$\eta = \frac{\frac{W}{g}(\bar{r})^2\ddot{\theta} + Wd.\sin\theta + \left(\kappa \times B \times s_{mob} + k\eta_{eq} \times \dot{\theta}\right)}{\frac{1}{2}\left[q_b + p(t)\right] \times B^2} + \frac{\frac{1}{2}\left[q_b + p(t)\right] \times B^2 + \mu_{dyn}N}{\frac{1}{2}\left[q_b + p(t)\right] \times B^2} = 1 \qquad (4)$$

Where η_{eq} is the equivalent viscous damping converted from Coulomb friction damping during sliding motion; r is the radius of gyration, equal to the square root of ratio J/F; kB = R is the radius of the sliding radii; k is the conversion constant from R to the width of the footing B. The equation (4) can write as below:

$$\frac{W}{g}(\bar{r})^2\ddot{\theta} + Wd.\sin\theta + \left(\kappa \times B \times s_{mob} + k\eta_{eq} \times \dot{\theta}\right) + \tfrac{1}{2}\left[q_b + p(t)\right] \times B^2 + \mu_{dyn}N = \tfrac{1}{2}\left[q_b + p(t)\right] \times B^2 \qquad (5)$$

This is a second-order ordinary differential equation of equilibrium for the structure. The equation can be shortened as:

$$\ddot{\theta} + A\dot{\theta} + \beta\theta = \frac{1}{2Ma^2}\left[p_{static}(1+\xi) - s_o\psi(t)\right] \qquad (6)$$

in which, ξ is fraction of dynamic contact pressure, as percentage of static contact pressure;

$$A = \frac{g}{W}\frac{\eta_{eq}}{(r)^2};\ \beta = \frac{d}{(\bar{r})^2};\ D = \frac{1}{2M}\frac{B^2}{\bar{r}^2} = \frac{1}{2M\alpha^2}$$

RESULTS

A Simulink diagram for solving the different equation (6) is developed as in the Figure 5 and Matlab toolbox in Figure 6 below:

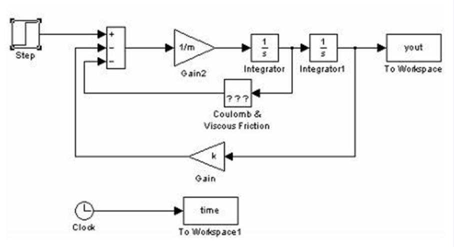
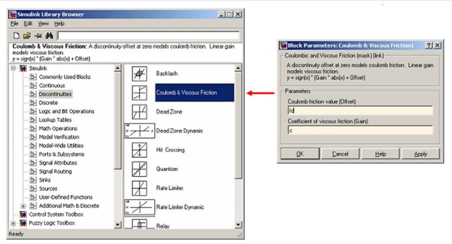


Figure 5: The conceptual diagram of Simulink for the problem.

To solve this problem in general, assuming all terms on the right-hand side are predictable and constant. For the mobilized shear strength s_{mob}, in the right-hand side of the equation (6) (i.e., ψ(t)) is kept to be constant; this is explanatory that in the specific case where the soil is a cohesive one, the shear strength is time-invariant, or ψ(t)=1.

Nevertheless, the equation (6) is in fact a non-linear second-order differential equation, or the governing equation for rotational angle of the structure subjected to vibration. Angular response of the solutionis plotted in Figure 7, nearly equal to 0.23 rad or approximately equal to 13 degrees. This result should be checked by comparing it with some other kinds of foundation.


Coulomb & Viscous Friction: A discontinuity offset at zero models coulomb friction. Linear gain models viscous friction.
y = sign(x) * (Gain * abs(x) + Offset)

Coulombic and Viscous Friction (mask) (link)

A discontinuity offset at zero models coulomb friction. Linear gain models viscous friction.
y = sign(x) * (Gain * abs(x) + Offset)

Parameters

Coulomb friction value (Offset): fd

Coefficient of viscous friction (Gain): c

Figure 6: Matlab-Simulink menu in which the Coulomb friction and viscous damping are assigned as Gain in the diagram.

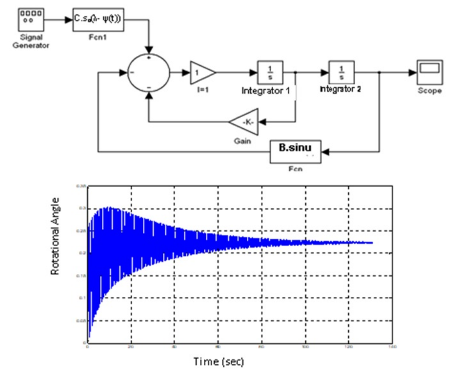


Figure 7: Solution obtained by the Matlab-Simulink

A NUMERICAL MODEL IN FAR FIELD FOR VALIDATION

For checking the solution obtained by the abovementioned mathematical model, a finite element model is studied. The assumption to be validated is that there is a differential settlement of an existing receiver footing resting on a cohesive soil foundation due to the dynamic effects propagating in the far field. As the problem of pile driving is the typical case of the dynamic effects[2], a finite element model in Plaxis is created as in Figure 8. A two-layered soil foundation has properties as described in Table 1. The existing foundation rests on a cohesive soil of the first layer.

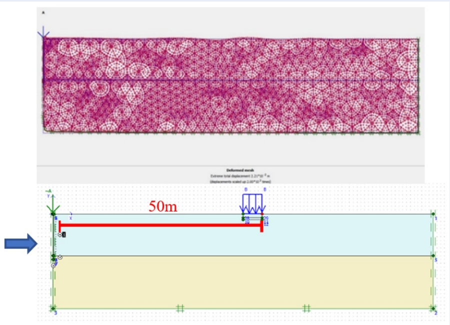


Figure 8: A finite element model for comparison purposes.

The source of excitation is a low-frequency impact load with the constant amplitude of load, being 50 kN, and the variety of frequency as in Table 2. A charge with the density p = 20 kPa is uniformly distributed over an area of 5 meters in diameter of the receiver footing, which is lying at the depth of 1 meter from the ground surface. The settlement without any dynamic effects in both cases is S_{static}= 7.912 mm, and the additional settlement due to vibration would be some percentage as compared to static ones.

Results in Table 2, which the settlements are measured at the center of the existing receiver footing. By investigating the wider distance, for instance 50m as in the Figure 8.

Results shown in Figure 9 implies that under a vibration caused by construction activity (i.e., low-frequency vibration) the vertical displacements U_y at two points and a depth 1m below the ground surface tend to be unequal, *-2 .32e-3* mm and *-1.15e-3* mm, resulting in an inclination of 6 degrees. This result from the numerical model confirms there is a rotation, originating from the unequal additional settlement (i.e., differential settlement), and partially due to the vibrational response of the receiver footing.

DISCUSSION

In case the right-hand side of the governing equation is a time-dependent one, or $\ddot{\theta}+\omega^2\theta = f(t)$, and the initial condition is $\theta = 0$ and $\dot{\theta} = 0$ the solution of the equation could be:

$$\theta(t) = \frac{F_o}{k^2} = \frac{1}{\left(1-\frac{\bar{\omega}}{\omega}\right)}\left[\sin(\bar{\omega}t) - \frac{\bar{\omega}}{\omega}\sin(\omega t)\right] \quad (7)$$

where ω, is the natural and angular frequency of the system and excitation, respectively; F_o is the amplitude of the excitation force. This solution is a sinusoidal response; due to damping, the amplitude is gradually decreased. As such the solution by Simulink is explanatory. With Simulink, once the conventional mathematical model is solved, the researcher can easily change the input style of vibration, for instance, perturbation as a summation of several sinusoidal waves, converting viscous damping to Coulomb friction, etc. Nevertheless, the sliding mass is not a rigid body. It is predictable the rotational angle could be larger than that of calculation.

The contact pressure that varies concerning the time p(t), its general equation could be determined as follow:

$$p(t) = \frac{p}{2m\omega^2}\left[1+\cos(\omega t)+\sin(\omega t)\right] \quad (8)$$

in which m is the mass of the structure (this weight is uniformly distributed over the area of footing). The time-dependent contact pressure would make the problem much more complicated and the non-linear analysis is beyond the scope of this paper.

Dynamic effects due to construction activities originate from low-frequency surface waves, traveling through the soil medium, and stirring the receiver footing. For reducing the damages due to the negative effects of the vibration, it is necessary to screen the vibration by trench or some specific measure of hardening the medium of wave propagation.

Coulomb friction damping developed along the sliding mass can be derived from the viscous damping by converting the friction damping to the viscous damping.

CONCLUSION

Vibration is a complicated topic that has not been thoroughly studied. Unlike earthquake that emits very large energy, the dynamic effects caused by construction activities have serious consequences.

This paper uses an approach that inherits existing studies on the dynamic behavior of strip foundations (Length L>7 Width); by analyzing the static and dynamic forces acting on the foundation, and the cohesive soil underlaying the foundation, an analytical equation of equilibrium of footing is established and solved by using the Simulink Tool of Matlab. The solution is the time-dependent rotational angle θ(t) which tends to converge to a constant value, equal to 0.22 radians or 13 degrees. This is theoretically solved and agreed partially with that of the numerical model Plaxis, which takes the effects on far field into account.

Table 1: **Properties of soil foundation**

Layer	Specification	Thickness	Cohesion (kPa)	φ^O	E (kPa)
1	Soft Clay (viscous)	11	12	3	1.25e4
2	Sand	14	1	31	5e4

Table 2: **Results by FEM modelabout additional settlement**

Frequency (Hz)	Static settlement (mm)	Additional settlement (mm)
1	7.978	0.166
5	8.279	0.467
10	8.098	0.286

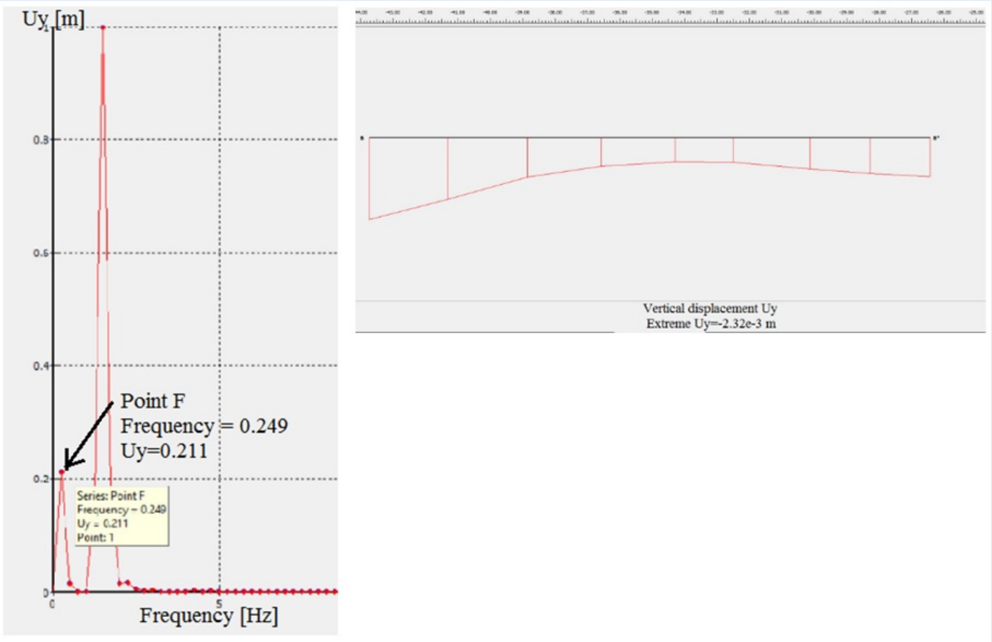


Figure 9: A finite element model for comparison purposes indicates the low-frequency response and displacements

This leads to a suggestion that most of the problems in Engineering Mechanics could be modeled and solved efficiently by Matlab-Simulink Toolbox. It facilitates engineers to integrate with other powerful and advanced tools of Matlab, for instance, Artificial Neural Network, Wavelet Toolbox, etc. so that scientists and engineers could tackle more advanced problems in manufacture and real practice.

CONFLICT OF INTEREST

Author confirms there is no conflict on the publication of this paper.

AUTHOR’ CONTRIBUTION

Author is the principal investigator of this paper.

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Tạp chí Phát triển Khoa học và Công nghệ – Engineering and Technology,4(SI2):SI91-SI98

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Bài nghiên cứu

Áp dụng Simulink vào giải phương trình vi phân xác định góc nghiêng của căn nhà đặt trên nền nhớt bị rung động

Dương Hồng Thẩm[*]

TÓM TẮT

Ảnh hưởng động lên công trình là những chủ đề phức tạp cả về lý thuyết và thí nghiệm. Bài báo này nhắm đến tìm hiểu sâu hơn về ảnh hưởng của sóng lên ứng xử của một móng nhận trong vùng lân cận nguồn dao động. Bài báo thiết lập phương trình cai quản kết cấu bị rung, nằm trên một nền nhớt. Bằng cách liệt kê tất cả những chuyển động xoay xảy ra cho móng, bao gồm mô men trượt xoay, các mô men chống trượt xoay và cân bằng, một phương trình vi phân được thiết lập. Đặc trưng nhớt vốn dựa trên những kết quả thí nghiệm trong phòng được xét đến, và phương trình cân bằng động lực của tất cả mô men nói trên được biến đổi để ra được một phương trình vi phân thường cấp 2 hệ số hằng số. Ngoài cách giải giải tích được giải bằng phương pháp thông thường, một sơ đồ Matlab Simulink được tạo ra nhằm giải phương trình đó. Kết quả chỉ ra rằng một giá trị góc xoay tại đó công trình bị rung động sẽ đạt đến trong suốt quá trình rung động trong một thời gian nhất định, Lời giải này có thể giải thích cho những kiểu hư hỏng nhà cửa xây dựng bên cạnh những con đường, bị kích thích và dao động dưới ảnh hưởng động gây bởi chuyển động của xe cộ.

Một mô hình Phần tử hữu hạn, Plaxis 2D được dùng để kiểm chứng tính đúng đắn của ứng xử móng nhận, nhưng xem xét ảnh hưởng trên miền xa.. Nền đất rất chịu nén, chịu rung động ở những tần số lực kích thích khác nhau. Dịch chuyển của những chất điểm trên miền tự do của bề mặt đất ở khoảng cách 30, 40 và 50 m cách nguồn gây rung đã dẫn đến sự lún tăng thêm của móng nhận. Và ứng xử chuyển dịch của kết cấu được phân tích cả miền thời gian và miền tần số. Lợi ích của việc áp dụng Matlab Simulink cho phân tích các bài toán riêng của cơ kỹ thuật được đề xuất.

Từ khoá: Nền nhớt, Đáp ứng dịch chuyển, Phương trình cai quản, Công cụ Simulink của Matlab

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