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Research article

Dynamic finite element analysis of microbeams under a moving mass based on refined higher-order shear deformation theory

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ABSTRACT

The dynamic behavior of microbeams with rectangular cross section under action of a moving mass is studied in the present paper in the framework of a refined higher-order shear deformation beam theory. The modified couple stress theory (MCST) with only one additional scale parameter is adopted to describe the influence of the microsize effect on the dynamic response of the microbeams. A finite element formulation with ten degrees of freedom is formulated and used to establish the discretized equation of motion for the microbeams. The formulation, taking into account the influence of the inertial effect, the Coriolis and centrifugal forces resulted from the mass moving, is derived from the expressions of the elastic and kinetic energies of the microbeams. The accuracy and efficiency of the derived formulation are confirmed by comparing the result obtained in the present work with the published data. The dynamic response of the microbeams with simply supported ends, such as the curves for dimensionless mid-span deflection-moving time relationship, the dynamic magnification factors (DMFs) and the thickness distribution of stresses are assessed by using an implicit Newmark method. The obtained numerical results reveal that the material length scale parameter which is introduced in the MCST has an important role on the dynamic behavior of the microbeams, and the DMF obtained from the theory incoporating the MCST is considerably lower than that using the conventional beam theory. It is also shown that the amplitude of both the axial stress and shear stress is considerably decreased by the increase of the material length scale parameter. A numerical study is carried out to highlight the influence of various parameters such as the moving mass velocity and the the ratio of the total beam length to its height on the dynamic response of the microbeams.

Key words: Microbeam, refined higher-order shear deformation beam theory, MCST, moving mass, finite element dynamic analysis

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INTRODUCTION

In recent years, beam-type microstructures, whose characteristic dimensions are measured in a micrometer scale, have wide application in the micro-electro-mechanical system (MEMS)[1]. Therefore, studies on the behavior of microbeams are of great interest. Theories of the classical continuum mechanics, however, cannot model the size dependence of the deformation behavior in the microbeam due to the lack of a material length scale parameter[2–4]. The classical couple stress theory with four material length scale parameter proposed in 60s of the last century can model the size effect the size effect faces difficulties in implementation. Yang et al.[2] modied the couple stress theory by reducing the scale parameter from four to only one, and this theory, named as modified couple stress theory (MCST), is widely used by researchers to study the behavior of microstructures. Regarding the microbeams, based on MCST and classical beam theory, Park and Gao[3] studied bending of a cantilever beam. They concluded that the size effect in the micron scale was significant in the beam bending problem. Ma et al.[4] extended the work by Park and Gao[3] to the bending and free vibration analysis of Timoshenko microbeam with simply supported ends. Kahrobaiyan et al.[5], Dehrouyeh-Semnani and Bahrami[6] adopted Timoshenko beam theory and CMST to develop beam elements for bending analysis of microbeam. Their numerical finding showed that the maximum deflections are overestimated by ignoring the influence of size effect.

The moving load/mass problem has attracted great attention from researchers due to its practical application. Therefore, it is crucial to understand the mechanical behavior of structures excited by moving loads. The mechanical behavior of the beam under moving loads has been carried out by various methods such as the modal superposition method[7], Galerkin method[8], Fourier transform technique[9], Lagrange method[10], the finite element method[10–12]. The above studies are performed with traditional beams based on the Euler-Bernoulli/Timoshenko beam the-

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ory, where the velocity of the moving load is mostly considered to be constant. Some recent reports focus on small-scale structures excited by the moving load. Kiani and Mehri[13] investigated the vibration of nanotubes excited by a moving nanoparticle in the framework of the nonlocal continuum theory of Eringen. Their investigation showed that the scale parameters and the moving nanoparticle velocity are important factors that significantly influence the maximum dynamic deflection of the nanotube structures. Based on the nonlocal Euler-Bernoulli beam theory, Şimşek[14] investigated the dynamic responses of a simply supported single-walled carbon nanotube subjected to a moving harmonic load. The dynamic behavior of the Timoshenko microbeam under a moving load was studied by Esen[15] using a MCST based finite element procedure. The author developed a simple finite element formulation with only four degrees of freedom by using the solution of the homogeneous equilibrium equations as the shape functions for study the influence of material size parameter and the load velocity on the beam dynamics. Esen et al.[16] used the same method in Esen[15] to investigate the vibration behavior of perforated microbeams traversed by moving load/mass. The parabolic shear deformation theory was employed in combination with MCST by Zhang and Liu[17] to study the forced vibration of porous microbeam excited by a moving harmonic load. The material of the beam is assumed to be graded in the thickness direction by a power-law distribution.

In this paper, the dynamic response of microbeams traversed by a moving mass is studied in the framework of the higher-order shear deformation beam theory and the finite element method. The MCST is adopted to model the size effect of the microbeams. Energy expressions for the beams and the moving mass are derived and used to derive a finite element formulation with ten degrees of freedom. The discretized equation of motion is established and solved by a direct integraation Newmark method. Numerical investigations are presented to show the effects of various factors such as the material length scale parameter, the moving mass velocity as well as the beam geometry on the dynamics of the microbeams.

THEORETICAL FORMULATION

Figure 1 shows a microbeam with length L , simply supported ends subjected to a moving mass m. In the figure, the beam cross section is considered to be rectangular with sizes ($b \times h$), and x_m is the current abscissa of the mass m, measured from the left support. It is asumed that the velocity of the mass m is unchanged during the mass moving from the left end to the right end of the beam. In addition, the mass is always in contact with the beam during it is on the beam. The plane (x,y) of the Cartesian coordinate system (x,y,z) in Figure 1 is identical to the beam mid-plane, while the z-axis directs upward.

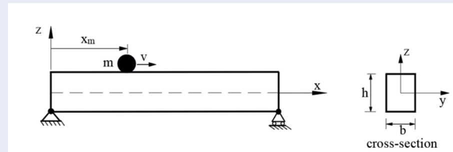

Figure 1: A simply supported microbeam under a moving mass

According to the refined higher-order shear deformation theory,[18] the displacements, $u_x(x,z,t)$ and $u_z(x,z,t)$, in the x- and z-direction, respectively, of a point in the beam are of the forms

$$\begin{aligned} & u_x(x,z,t) = \\ & u(x,t) - z w_{b,x}(x,t) - f(z) w_{s,x}(x,t), \\ & u_z(x,z,t) = w_b(x,t) + w_s(x,t) \end{aligned} \quad (1)$$

where is the displacement in x-direction of the point on the x-axis; w_b and w_s are, respectively, bending part and shear part of the transverse displacement; t is the time variable, and $f(z)$ is the section shape function of the axial displacement, and it is of the form

$$f(z) = \frac{4z^3}{3h^2} \quad (2)$$

The subscript comma in Eq. (1) and hereafter indicate the derivative with respect to the variable which follows.

The microsize elfect is modelled herein by using the MCST of Yang et al.[2]. In this regard, the elastic energy of the microbeam is given by

$$U = \frac{1}{2} \int_v \left(\sigma_{ij} \varepsilon_{ij} + m_{ij} \chi_{ij} \right) dV, \ i,j = x,y,z \quad (3)$$

In the above equation, 0the sum is performed on repeated indices; V is the volume of the beam; σ_{ij}, ε_{ij}, m_{ij} and χ_{ij} are, respectively, the components of the stress and strain tensors, deviatoric part of the couple stress tensor and symmetric curvature tensor. These componeents are defined á follows

$$\begin{aligned} & \sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2G\varepsilon_{ij}, \ \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \\ & m_{ij} = 2l^2 G \chi_{ij}, \ \chi_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right) \end{aligned} \quad (4)$$

with u_i (i=x,y,z) is the displacement in the i direction; l is a material length scale parameter; $\lambda = \frac{EV}{(1+v)(1-2v)}$

and $G=\frac{E}{2(1+v)}$ are Lame's constants, in which E is elastic modulus, and υ is Poisson's ratio; θ_i is the components of the rotation vector and it is given by

$$\theta=\frac{1}{2}e_{ijk}u_{k,j} \tag{5}$$

where e_{ijk} is a Levi-Civita system that allows to determine the sign in the permutation in tensor analysis. From Eq. (1) and Eqs. (4)-(5), the elastic energy U in Eq. (3) can be recast in the form

$$\begin{aligned}U=&\\ &\frac{1}{2}\int_0^L\int_A\left(\sigma_{xx}\varepsilon_{xx}+2\sigma_{xz}\varepsilon_{xz}+2m_{xy}\chi_{xy}\right)dA\\ &=\frac{1}{2}\int_0^L(A_{11}u_{,x}^2-A_{12}u_{,x}w_{b,xx}+A_{22}w_{b,xx}^2-\\ &2A_{13}u_{,x}w_{s,xx}+2A_{23}u_{,x}w_{s,xx}+A_{33}w_{s,xx}^2\\ &+2B_{11}w_{b,xx}^2+2B_{12}w_{b,xx}w_{s,xx}+B_{22}w_{s,xx}^2\\ &+D_{11}w_{s,x}^2)dx\end{aligned} \tag{6}$$

in which A=bxh is denotes the area of the beam cross section; A_{11}, A_{12}, A_{22}, A_{13}, A_{23}, A_{33}, B_{11}, B_{12}, B_{22} and D_{11} are the rigidities of the microbeam which are defined as

$$\begin{aligned}&(A_{11},A_{12},A_{22},A_{13},A_{23},A_{33})\\ &=b(\lambda+2G)\int_{-h/2}^{h/2}\left[1,z,z^2,f(z),zf(z),f^2(z)\right]dz\\ &(B_{11},B_{12},B_{22})\\ &=\frac{bl^2G}{2}\int_{-h/2}^{h/2}\left\{1,\left[1+f_{,z}(z),\frac{1}{2}\left[1+f_{,z}(z)\right]^2\right]\right\}dz\\ &D_{11}=bG\int_{-h/2}^{h/2}\left\{\left(1-\frac{4z^2}{h^2}\right)+\left[\frac{lf_{,zz}(z)}{2}\right]^2\right\}dz\end{aligned} \tag{7}$$

The kinetic energy T of the microbeam is of the form

$$T=\frac{1}{2}\int_0^L\int_A\rho\left(\dot{u}_x^2+\dot{u}_z^2\right)dAdx \tag{8}$$

where ρ denotes the mass density, and the over-dot in the equation and hereafter denotes the partial derivative with respect to the time variable. Using Eqs. (1) and (2), the kinetic energy in (8) can be rewritten in the following form

$$\begin{aligned}T=&\frac{1}{2}\int_0^L\{I_{11}\left[\dot{u}^2+\left(\dot{w}_b+\dot{w}_s\right)^2\right]-2I_{12}\dot{u}\dot{w}_{b,x}\\ &+I_{22}\dot{w}_{b,x}^2-2I_{13}\dot{u}\dot{w}_{s,x}+2I_{23}\dot{w}_{b,x}\dot{w}_{s,x}\\ &+I_{33}\dot{w}_{s,x}^2\}dx\end{aligned} \tag{9}$$

where I_{11}, I_{12}, I_{22}, I_{13}, I_{23} and I_{33} are the mass moment coefficients of the beam, and they are defined as

$$\begin{aligned}&(I_{11},I_{12},I_{22},I_{13},I_{23},I_{33})\\ &=b\rho\int_{-h/2}^{h/2}\left[1,z,z^2,f(z),zf(z),f^2(z)\right]dz\end{aligned} \tag{10}$$

The potential energy of the moving mass is given by [11]

$$\begin{aligned}&V_m=-\int_0^L\\ &\left[\left(mg-m\ddot{u}_z-2mv\dot{u}_{z,x}-mv^2u_{z,xx}\right)u_z(x,t)\right.\\ &\left.-m\ddot{u}u(x,t)\right]\delta\left(x_m-vt\right)dx\end{aligned} \tag{11}$$

In the above equation, g represents the gravity acceleration; $m\ddot{u}$ and $m\ddot{u}_z$ are the inertia forces in the axial and transverse directions, respectively; $2mv\dot{u}_{z,x}$ is the Coriolis force; $mv^2u_{z,xx}$ is the centrifugal forces; and $\delta(.)$ denotes the Dirac delta function.

SOLUTION METHOD

The present work employs the finite element method to compute the dynamic characteristics resulted from the moving mass. To this end, a finite element model, namely a two-node beam element is derived in this section. Denoting l_e as the length of a generic element. The element has ten degrees of freedom with the vector of nodal displacement d of the form

$$\underset{10\times1}{d}=\left\{d_u\quad d_{w_b}\quad d_{w_s}\right\}^T \tag{12}$$

with d_u, d_{w_b} and d_{w_s} are, respectively, the nodal displacement vectors for the u, w_b and w_s. These vectors are represented as follows

$$\begin{aligned}&d_u=\left\{u_1\quad u_2\right\}^T,\\ &d_{w_b}=\left\{w_{b1}\quad w_{b1,x}\quad w_{b2}\quad w_{b2,x}\right\}^T\\ &d_{w_s}=\left\{w_{s1}\quad w_{s1,x}\quad w_{s2}\quad w_{s2,x}\right\}^T\end{aligned} \tag{13}$$

where u_i, w_{bi}, $w_{bi,x}$, $w_{si}$and $w_{si,x}$ (i = 1;2) are the nodal values of u, w_b, $w_{b,x}$, w_s and $w_{s,x}$ at the two nodes, respectively; In Eq. 9130 and hereafter the superscript "T" indicates the transpose of a vector or a matrix. Interpolations are now needed to introduce for the axial and transverse displacements as

$$u=Nd_u,\ w_b=Hd_{w_b},\ w_s=Hd_{w_s} \tag{14}$$

where $\mathbf{N}$=[N_1 N_2], $\mathbf{H}$=[$H_1H_2H_3$ H_4] with N_1 and N_2 are Lagrange polynomials, and H_1, H_2, H_3, H_4 are Hermite polynomials.

With the introduced interpolations, one can rewrite the elastic energy of the microbeam in Eq. (6) in the following matrix form

$$U=\frac{1}{2}\sum^{ne}d^Tk_ed \tag{15}$$

where ne is the number of elements needed to discretize the beam; and k_e is the stiffness matrix of the element, and it can be written in sub-matrices as

$$\underset{10\times10}{k_e}=\begin{bmatrix}k_{aa} & k_{ab} & k_{as}\\ (k_{ab})^T & k_{bb} & k_{bs}\\ (k_{as})^T & (k_{bs})^T & k_{ss}\end{bmatrix} \tag{16}$$

The sub-matrices $\mathbf{k}_{aa}$, $\mathbf{k}_{bb}$ and $\mathbf{k}_{ss}$ in Eq. (16) represent the element stiffness matrices resulted from the axial stretching, bending and shear deformation, respectively. These sub-matrices are defined as

$$\underset{2\times 2}{k_{aa}} = \int_0^{l_e} N_{,x}^T A_{11} N_{,x} dx,\ \underset{4\times 4}{k_{bb}} = \int_0^{l_e} H_{,xx}^T (A_{22} + 2B_{11}) H_{,xx} dx,\ \underset{4\times 4}{k_{ss}} = \int_0^{l_e} \left[H_{,xx}^T (A_{33} + B_{22}) H_{,xx} + H_{,x}^T D_{11} H_{,x} \right] dx \quad (17)$$

The submatrices $\mathbf{k}_{ab}$, $\mathbf{k}_{as}$ and $\mathbf{k}_{bs}$ in (16) are, respectively, the stiffness matrices due to the axial stretching-bending, axial stretching-shear, bending-shear coupling effects, and they have the following forms

$$\begin{aligned} \underset{2\times 4}{k_{ab}} &= -\int_0^{l_e} N_{,x}^T A_{12} H_{,xx} dx, \\ \underset{2\times 4}{k_{as}} &= -\int_0^{l_e} N_{,x}^T A_{13} H_{,xx} dx, \\ \underset{4\times 4}{k_{bs}} &= \int_0^{l_e} H_{,xx}^T (A_{23} + B_{12}) H_{,xx} dx \end{aligned} \quad (18)$$

Similarly, the kinetic energy of the microbeam defined by Eq. (9) can also be written in a matrix form as

$$T = \frac{1}{2} \overset{ne}{\Sigma} \dot{d}^T m_e \dot{d} \quad (19)$$

which the mass matrix of the element m_e can be rewritten in the form

$$\underset{10\times 10}{m_e} = \begin{bmatrix} m_{aa} & m_{ab} & m_{as} \\ (m_{ab})^T & m_{bb} & m_{bs} \\ (m_{as})^T & (m_{bs})^T & m_{ss} \end{bmatrix} \quad (20)$$

where

$$\begin{aligned} \underset{2\times 2}{m_{aa}} &= \int_0^{l_e} N^T I_{11} N dx, \\ \underset{4\times 4}{m_{bb}} &= \int_0^{l_e} \left(H^T I_{11} H + H_{,x}^T I_{22} H_{,x} \right) dx, \\ \underset{4\times 4}{m_{ss}} &= \int_0^{l_e} \left(H^T I_{11} H + H_{,x}^T I_{33} H_{,x} \right) dx, \\ \underset{2\times 4}{m_{ab}} &= -\int_0^{l_e} N^T I_{12} H_{,x}^T dx, \\ \underset{2\times 4}{m_{as}} &= -\int_0^{l_e} N^T I_{13} H_{,x}^T dx, \\ \underset{4\times 4}{m_{bs}} &= \int_0^{l_e} \left(H^T I_{11} H + H_{,x}^T I_{23} H_{,x} \right) dx \end{aligned} \quad (21)$$

The potential energy given by Eq. (11) can be written as follows

$$V_m = \overset{ne}{\Sigma} (\ddot{d}^T m_m \ddot{\mathbf{d}} + \dot{\mathbf{d}}^T c_m \dot{d} + d^T k_m d - d^T f_m) \quad (22)$$

with $\mathbf{m}_m$, $\mathbf{c}_m$ and $\mathbf{k}_m$ represent the element mass, damping and stiffness matrices caused by the influence of the inertia, Coriolis force and the centrifugal force of the mass; $\mathbf{f}_m$ is the element vector of the external load, and it is time dependent. The detail expressions of these matrices and vector are given below

$$\underset{10\times 10}{M_m} = m \begin{bmatrix} N^T N & 0 & 0 \\ 0 & H^T H & H^T H \\ 0 & H^T H & H^T H \end{bmatrix}_{x_e}, \quad (23)$$

$$\underset{10\times 10}{c_{\mathbf{m}}} = 2mv \begin{bmatrix} 0 & 0 & 0 \\ 0 & H^T H_{,x} & H^T H_{,x} \\ 0 & H^T H_{,x} & H^T H_{,x} \end{bmatrix}_{x_e}, \quad (24)$$

$$\underset{10\times 10}{k_m} = mv^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & H^T H_{,xx} & H^T H_{,xx} \\ 0 & H^T H_{,xx} & H^T H_{,xx} \end{bmatrix}_{x_e}, \quad (25)$$

and

$$\underset{10\times 1}{f_m} = mg \begin{bmatrix} 0 & H^T & H^T \end{bmatrix}_{x_e}^T \quad (26)$$

In Eqs. (23)-(26), the notation $[.]_{xe}$ indicates that the quantity inside the square brackets [.] is evaluated at x_e the current abscissa of the mass m. This abscissa is calculated from the left node of the element. It is worthy to note that the element matrices $\mathbf{m}_m$, $\mathbf{c}_m$, $\mathbf{k}_m$ and the force vector $\mathbf{f}_m$ are all zeros for all elements except for the element that currently contains the moving mass.

By asembling the element matrices and vector into the global ones, one can establish the discretized equation of motion for the vibration analysis of the microbeam under a moving mass in the following form

$$(M + M_m)\ddot{D} + C_m \dot{\mathbf{D}} + (K + K_m) D = F \quad (27)$$

In the above equation, $\mathbf{D}$, $\dot{D}$ and $\ddot{D}$ denote the structural vectors of nodal displacements, nodal velocities and nodal accelerations, respectively; $\mathbf{M}$, $\mathbf{M}_m$ are the structural mass matrices, resulted from assembling the element matrices m_e, $\mathbf{m}_m$, respectively; $\mathbf{C}_m$, $\mathbf{K}$, $\mathbf{K}_m$ and $\mathbf{F}$ are the structural matrices and vector, which are result of assembling the matrices $\mathbf{c}_m$, $\mathbf{k}_e$, $\mathbf{k}_m$ and $\mathbf{f}_m$ over the elements, respectively. In the present work, an implicit direct integration method, namely the average acceleration method of the Newmark family of methods with Newmark parameters β=1/4, γ=1/2 [19] is adopted in solving Eq. (27). This implicit method ensures the unconditional convergence of the numerical algorithm, and its details can be found in the textbook [19].

NUMERICAL RESULTS AND DISCUSSION

This section performs numerical investigations to study the dynamic behavior of the microbeam subjected by the moving mass. The beam considered herewith is made from Aluminum (Al) with the material data are as follows: E=70 GPa, ρ=2702 kg/m^3, υ=0.3. The geometrical parameters of the beam are defined as [17]: h=10 μm and b=1 μm. For the convenience of discussion, the dimensionless parameters represented the dynamic magnification factor (DMF - D_d), the mass ratio (r_m) and the mass velocity (β) are used

$$D_d = max\left(\frac{w(L/2,t)}{w_{st}}\right),$$
$$r_m = \frac{m}{\rho AL},\ \beta = \frac{v}{V_1},\ V_1 = \frac{L\omega_1}{\pi} \quad (28)$$

where w_{st}=$mgL^3/(48EI)$ is the static deflection of the beam under action of the load mg at the mid-span; ω_1 is the first natural frequency of the microbeam. The microbeam is assumed to be simply supported at both two ends.

The derived beam element should be validated to en ure its accuracy. In Table the fundamental frequency parameters of a macro beam obtained herein by setting l=0 are compared with the analytical solution obtained by Sina et al.[20]. The frequency parameters are calculated for a beam with L/h=(10, 30, 100). It can be observed from Table 1 that the frequency parameters obtained by the present method agrees well with the referenced data, regardless of the length-to-height ratio.

Table 2 compares the dynamic magnification factors of a macro beam (l=0) traversed by a moving load with various values of the speed parameter obtained in the present work with the finite element solution of Olsson[21]. It is necessary to note that the DMFs obtained by Olsson[21] are based on the classical Euler-Bernoulli beam theory. A good agreement between the result of the present paper with that of Olsson[21] can be noted from Table 2.

Table shows 3 convergence of the present beam element in computing DMFS of the microbeam subjected to the moving mass for r_m=0.5, β=0.1 and different values of the dimensionless material length scale parameter l/h aa well as the length-to-height ratio L/h. One can see from the table that the convergence of the present element is relative fast, and the used of 14 elements ensures the convergence, irrespective of the values of l/h and L/h. Because of this convergence, 14 elements are used for all the computations below.

The effects of the mass ratio r_m, dimensionless microscale parameter l/h and length-to-height ratio L/h on the dynamic magnification factor D_d of microbeam are given in Table 4 for a velocity parameter β=0.2. It can be seen from Table 4 that the factor D_d increases by increasing r_m and decreasing l/h, regardless of the length-to-height ratio L/h. The effect of the length-to-height ratio L/h on the factor D_d is more signification with values of L/h smaller than 20, regardless of the mass ratio and scale parameter.

The curves of dimensionless moving mass time versus the mid-span deflection of the microbeam are depicted in Figure 2 for L/h=20, r_m=0.5 and various values of the velocity ratio β and microscale parameter l/h. It is evident that both speed parameter β and microscale parameter l/h have a significant effect on the mid-span deflection. The mid-span deflection of the microbeam declines when increase the value of microscale parameter l/h, irrespective of the speed ratio. Thus, the traditional beam theory which ignores the size effect results in an overestimation of the dynamic deflection of the microbeam. The figure also shows that the maximum mid-span deflection of the beam is larger when it is under a mass moving with a higher velocity, regardless of the value of the microscale parameter l/h.

The variation of the DMF D_d with the moving mass velocity parameter β of the microbeam is displayed in Figure 3 for a length-to-height ratio L/h=20 and different values of the mass ratio r_m and the microscale parameter l/h. The dynamic magnification factor D_d in the figure undergoes a repeatedly increasing and decreasing period when increasing the velocity parameter β, and it then approaches a maximum value. The influence of the material length microscale parameter l/h on the dynamic response is clearly seen again from Figure 3a, where the factor D_d is decreased by increasing scale parameter l/h, regardless of the speed parameter. Besides, as expected, we can observe that the DMF D_d increases when increase the mass ratio r_m (Figure 3b).

The length-to-height ratio L/h versus the dynamic magnification factor D_d of the microbeam is shown in Figure 4 for different values of the microscale parameter l/h, the mass ratio r_m and for a velocity parameter β=0.2. The influence of the length-to-height ratio on the DMF, as can be seen from Figure 4a, tends to be less significant when the microsize effect is taken into consideration. Since the length-to-height ratio represents the influence of the shear deformation on behavior of the beam, the ignoring the microsize effect is not only overestimated the dynamic deflection, but it is also overestimated the shear deformation effect of the microbeam. The mass ratio, as seen from Figure 4b, is hardly changed the relation between the DMF with the length-to-height ratio, and the influence of the length-to-height ratio on the dynamics of the microbeam is more significant for L/h<20.

The distribution of axial and shear stresses in the thickness direction of the microbeam is respectively presented in Figure 5 and Figure 6 for L/h=20, r_m=0.5, β=0.1 and various values of the dimensionless scale

Table 1: Comparison of fundamental frequency parameters of a macro beam

Source	L/h=10	L/h =30	L/h =100
Sina et al. [20]	2.797	2.843	2.848
Present	2.7977	2.8432	2.8486

Table 2: Comparison of dynamic magnification factor of a macro beam

Sourse	B			
	0.125	0.25	0.5	1
Olsson [21]	1.121	1.258	1.705	1.548
Present	1.1300	1.2661	1.7162	1.5640

Table 3: Convergence of the beam element in computing DMF of microbeam (β =0.1, rm=0.5).

l/h	L/h	ne=4	ne=6	ne=8	ne=10	ne=12	ne=14
0.1	10	0.7935	0.7986	0.7977	0.7972	0.7971	0.7971
	50	0.7679	0.7720	0.7715	0.7713	0.7712	0.7712
	100	0.7671	0.7713	0.7707	0.7705	0.7705	0.7705
0.5	10	0.4363	0.4388	0.4384	0.4382	0.4381	0.4381
	50	0.4274	0.4297	0.4294	0.4293	0.4293	0.4293
	100	0.4272	0.4295	0.4291	0.4290	0.4290	0.4290

Table 4: Dynamic magnification factor of the microbeam for β=0.2.

		L/h			
r_m	l/h	10	20	30	50
0.25	0.25	0.7068	0.6918	0.6886	0.6870
	0.5	0.4589	0.4513	0.4497	0.4491
	1	0.1908	0.1888	0.1884	0.1883
0.5	0.25	0.7505	0.7336	0.7303	0.7287
	0.5	0.4863	0.4786	0.4770	0.4763
	1	0.2022	0.2003	0.1999	0.1997
1	0.25	0.8402	0.8207	0.8170	0.8150
	0.5	0.5440	0.5353	0.5336	0.5327
	1	0.2262	0.2240	0.2235	0.2233

parameter l/h. The stresses in these figures are calculated at the time when the mass m arrives at the beam mid-span. Both the axial stresss and shear stress are normalized by a compressive stress $\sigma_0=mg/(bh)$, that is $\sigma_{xx}{}^* = \sigma_{xx}(L/2, z)/\sigma_0$, and $\tau_{xz}{}^*=\tau_{xz}(0, z)/\sigma_0$. As seen from these figures, both the axial and shear stress are symmetric to the mid-plane of the microbeam, regardless of the value of the microscale parameter l/h. In addition, all stresses considered herein are significantly influenced by the microscale parameter, and an increase of the microscale parameter l/h results in a decreases of the maximum axial stress and shear stress.

CONCLUSION

The dynamics of the microbeams subjected to a moving mass has been investigated in the present work in the framework of the refined higher-order shear deformation theory and the finite element method. The MCST containing only one microscale parameter is

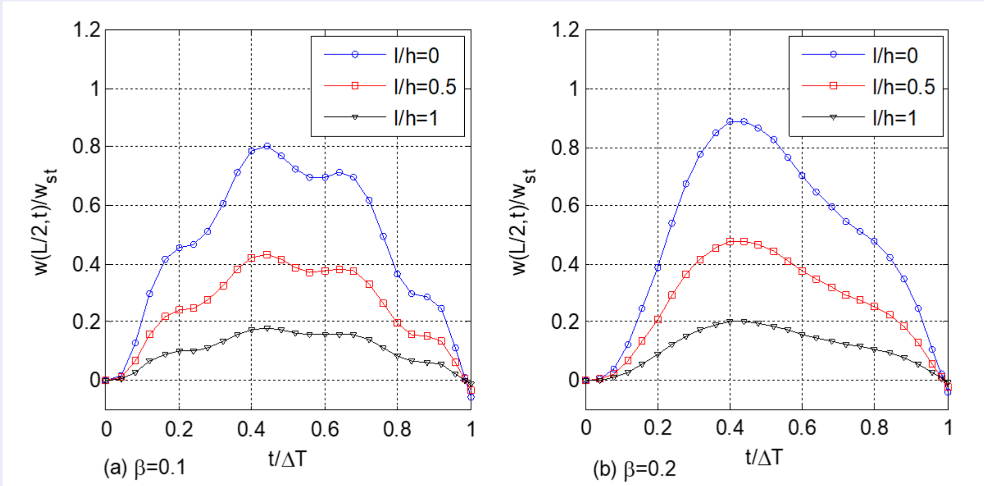

Figure 2: Dimensionless moving mass time versus mid-span deflection of the microbeam for L/h=20, r_m=0.5

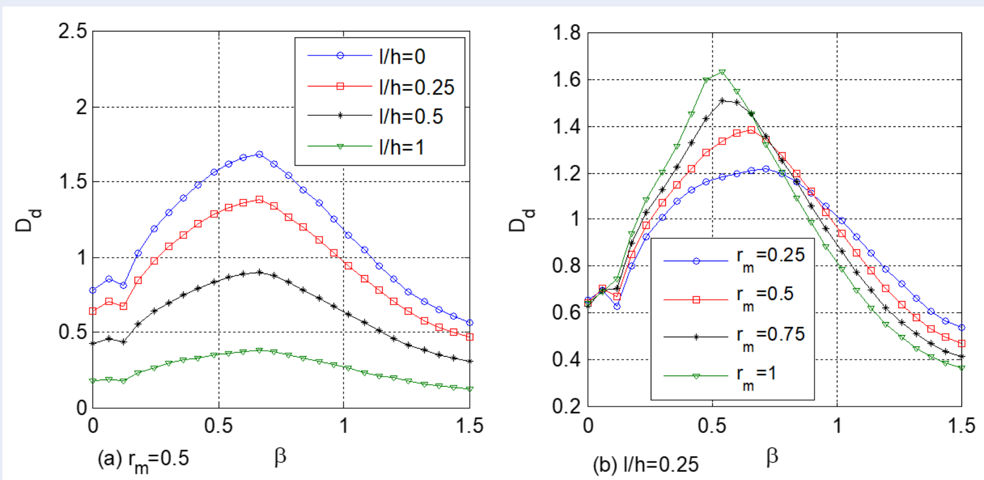

Figure 3: Variation of DMF with velocity parameter of the microbeam for L/h=20.

adopted to model the size effect of the beams. The discretized equation of motion for the microbeams was established by using a ten degrees of freedom finite element formulation. Using the direct integration Newmark method, the dynamic response such as the curves for dimensionless time-deflection relation, the dynamic magnification factors as well as the stresses of the microbeam were computed. The obtained numerical re ults show that the microsize effect has an important role on dynamics of the microbeam, and the classical beam theories which ignore the size effect leads to an overestimation of the dynamic magnification factors and the dynamic deflections of the microbeam. Investigations have been carried out to highlight in detail the effects of various parameters such as the mass velocity parameter, the mass ratio, the beam length-to-height ratio on the dynamic behavior of the microbeam. We would like to note that though the numerical investigations have been presented in this paper for a microbeam with simply supported ends only, the microbem model and the finite element formulation of the present work are capable to compute dynamics of microbeams under a moving mass with other boundary conditions as well.

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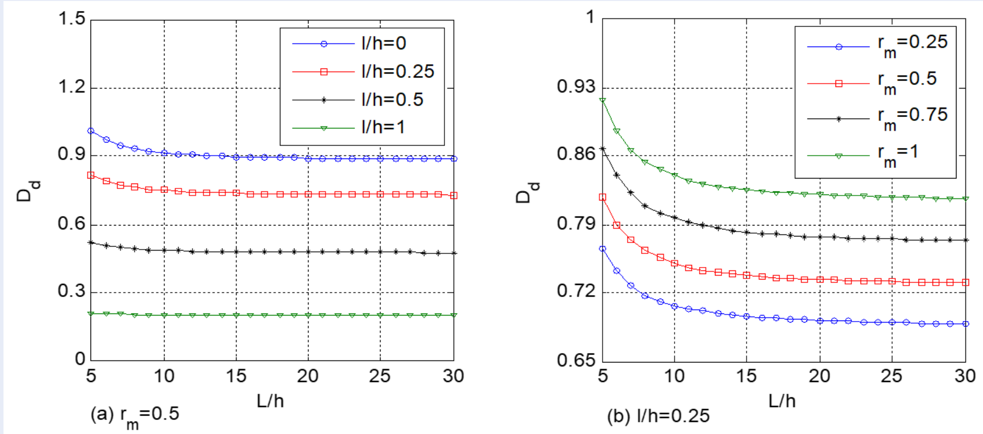

Figure 4: Length-to-height ratio versus the DMF of the microbeam for β=0.2

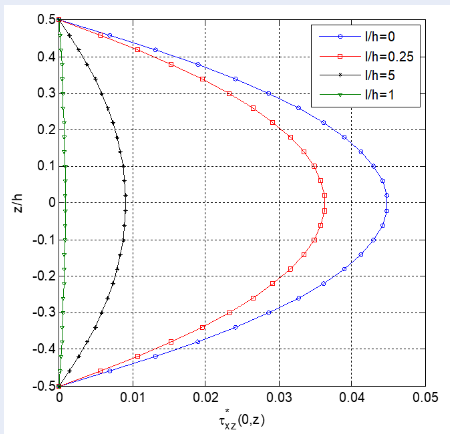

Figure 6: Thickness distribution of shear stress of the microbeam for L/h=20, r_m=0.5, β=0.1

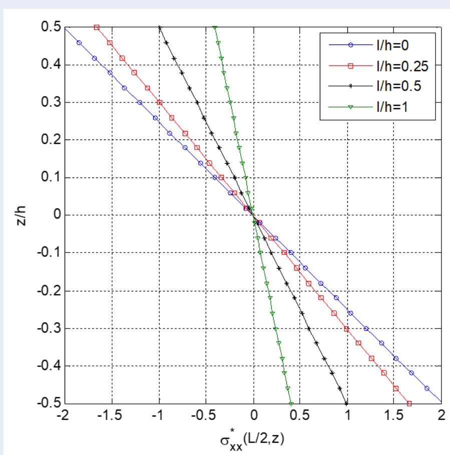

Figure 5: Thickness distribution of axial stress of the microbeam for L/h=20, r_m=0.5, β=0.1

CONFLICT OF INTEREST

There is no conflict of interest.

CONTRIBUTION OF EACH AUTHOR

Vu Nam Pham: formal analysis, validation, writing draft.

An Ninh Thi Vu: conceptualization, software, review and editing, supervision.

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Open Access Full Text Article

Phân tích phần tử hữu hạn động lực học của dầm micro chịu khối lượng di động sử dụng lý thuyết biến dạng trượt bậc cao cải biên

Phạm Vũ Nam[1], Vũ Thị An Ninh[2,*]

TÓM TẮT

Bài báo này nghiên cứu ứng xử động lực học của dầm micro với mặt cắt ngang hình chữ nhật chịu tác động của khối lượng di động trong khuôn khổ lý thuyết biến dạng trượt bậc cao cải biên. Lý thuyết ứng suất cặp cải biên (MCST) chứa một tham số kích cỡ được sử dụng để mô tả ảnh hưởng của hiệu ứng kích thước micro lên đáp ứng động lực học của dầm. Công thức phần tử hữu hạn mười bậc tự do được xây dựng và sử dụng để thiết lập phương trình chuyển động rời rạc cho dầm micro. Công thức, có tính đến ảnh hưởng của lực quán tính, lực Coriolis và lực ly tâm do khối lượng di động, được xây dựng từ các biểu thức năng lượng biến dạng và động năng của dầm micro. Độ chính xác và tính hiệu quả của công thức đưa ra được khẳng định bằng cách so sánh các kết quả nhận được trong bài báo với các kết quả trong tài liệu đã có. Đáp ứng động lực học của dầm micro với biên tựa hai đầu như: đường cong giữa độ võng dầm không thứ nguyên – thời gian, hệ số động lực học (DMFs), sự phân bố ứng suất theo chiều dầy dầm được đánh giá bằng phương pháp Newmark ẩn. Các kết quả số thu được cho thấy tham số kích cỡ vật liệu, trong MCST, đóng vai trò quan trọng đối với ứng xử động lực học của dầm micro, và DMF nhận được từ lý thuyết kết hợp với MCST thấp hơn đáng kể so với DMF nhận được từ lý thuyết dầm truyền thống. Kết quả cũng chỉ ra rằng biên độ của ứng suất dọc trục và ứng suất trượt giảm đáng kể khi tăng tham số kích cỡ vật liệu. Nghiên cứu số được thực hiện để làm rõ ảnh hưởng của các tham số khác như vận tốc khối lượng di động và tỷ số giữa độ dài dầm và chiều cao tới đáp ứng động lực học của dầm micro.

Từ khoá: Dầm micro, lý thuyết biến dạng trượt bậc cao cải biên, MCST, khối lượng di động, phân tích phần tử hữu hạn động lực học

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