Maxwell model geotextile encased stone column in soft soil improvement

Pham Tien Bach, Vo Dai Nhat, Le Quan, Nguyen Viet Ky

ABSTRACT
In the field of geotechnical – soft soil improvement, the mathematical model or mechanical model is one of the important input parameters for the design calculations or studies. The determination of the appropriateness of the models has a great influence on the accuracy results of design and calculation as well as the sustainable stability of soft ground after improvement. On the contrary, the selection of inadequate calculation models will lead to increased costs of soft soil improvement, possibly even leading to the destabilization of the work and causing immense loss of people and property.

Recently, many projects major highway after construction design in use has not meet the requirements of the standard, leading to wasted money and time of individuals, organizations, and the state of post-treatment. Therefore, the research and application of using mathematical or mechanical models in accordance with the new soft soil improvement method will greatly help as well as add additional options for soft soil improvement in Vietnam. The soft soil deformation is not only related to load but also to load time. The change in stress and deformation of weak soil over time is called rheology, and in this study is the viscoelastic behavior.

From the above reasons, we try to apply a generalized Maxwell model to explain the viscoelastic behavior of a soft soil. In particular, the time-dependent behavior of a viscoelastic soft soil was represented by using the Maxwell rheological model. The Matlab programming code helps to solve numerically all the equation of the mathematical exhibition of the generalized Maxwell model results. We acknowledge that the generalized Maxwell model is superior in demonstrating the results probably show that this is one of the effective models to predict the behavior of soft soils in ground improvement with GEC.

Key words: Maxwell model, Geotextile encased stone column (GEC), Soft soil improvement, Viscoelastic behavior

INTRODUCTION
The nature of soft soil such as low bearing capacity, high compressibility and low permeability cause difficulties for designing/constructing geotechnical engineering structures such as embankments, petroleum refinery, thermal power plants and infrastructure on them. Encased column is a successful well-known method that reduce the soft soil problem and can be improve ground and bearing system in very soft soils with undrained shear strength \( c_u < 5 \text{kN/m}^2 \). The load and the loading time exacerbate the soft soil deformation. Rheological properties describe the variety of soft soil stress and strains during the time such as creep, stress relaxation, long-term strength, and after effect elasticity \(^1\). Next the theory of the rheological model will be described and then a generalized Maxwell model is to constitute the mechanical rheological properties of soft soil. An ordinary differential equation is generated from the designed model then numerically solved by a Matlab programming code. The relaxation modulus \( E(t) \) of the generalized Maxwell model and its application are discussed. Thereafter, this study includes a manual application of the generalized Maxwell model to the time-dependent settlements of GEC for improvement of soft soil.

METHODOLOGY
Theory of Maxwell Model and Generalized Maxwell Model
We begin with examining all the characteristics of rheological viscoelastic behavior. Then we can reconstruct the behavior of the soft soil system with changeable elements to account for the elasticity and viscosity. To identify the stress inside the soft soil, we need to measure the force of a soft soil.

\[
\sigma = \frac{F}{A}
\]  

Where \( \sigma \) represents the stress, \( F \) the force and \( A \) the surface of the cross section of soft soil perpendicular to the direction of the force.

We also assume that the behavior of soft soil as a linear-elastic soil as written in Hooke’s Law (equation (2)).

\[ \sigma = E \cdot \varepsilon \]  

(2)

Where \( \sigma \) describes the stress of the spring in MPa, \( E \) the elastic modulus in MPa and \( \varepsilon \) the applied strain.

A simple model that describes such behavior would be an ordinary spring to which a strain is applied. A simple model could only illustrate the linear increase of the stress in the time, but after this time the relaxation does not appear at all. So, the process of relaxation must be considered into the model. In rheology, this phenomenon is called viscosity, and in a model, it is represented by a dashpot in series with a spring. This system is called a Maxwell element (Figure 1).

![Figure 1: a Maxwell element - A dashpot is placed in series with a spring to simulate viscous behavior.](image1)

To model viscous behavior of a soft soil, a dashpot with a known stress-strain relation is used. Stress of dashpot elements is related with strain differentiated by time \( t \), and the constitutive relation is time-dependent:

\[ \sigma_d = \eta \frac{ds}{dt} \]  

(3)

With \( \varepsilon \) is the strain, \( \sigma_d \) is the stress of the dashpot, \( \sigma_s \) is the stress of the spring and \( \eta \) is the viscosity of the soft soil.

To describe the appropriate behavior of this system a constitutive equation that accounts for both the strain of the dashpot as well as the strain of the spring is needed. The constitutive equations of the series model of spring and dashpot are as follows:

\[ \sigma_s = \sigma_d \Leftrightarrow \eta \frac{ds}{dt} = \eta \frac{ds_d}{dt} \]  

(4)

\[ \sigma + \frac{\alpha \eta}{(1 - \alpha)E} \cdot \frac{d\sigma}{dt} = \eta \frac{ds}{dt} \]  

(5)

By adding multiple Maxwell elements, a generalized Maxwell model, as shown in Figure 3, is built to approximate viscoelastic behavior of soft soil possible better.

![Figure 2: Stress relaxation](image2)

![Figure 3: Schematic of a generalized Maxwell model](image3)

With \( n \) the total number of Maxwell elements in the generalized Maxwell model, the mechanical properties are described by summation of the stress in each Maxwell element.

\[ \sigma_t = \sum_{i=1}^{n} \sigma_i \]  

(7)

**Model Description**

The behavior of geotextile encased stone column (GEC) on soft foundation soil has been idealized by the proposed foundation model (Figure 4) and assuming soft ground was supposed as a viscoelastic material. The paper focused on finding the suitability of the Maxwell model for soft soil with the simple analytical calculation of Raithel, M., Kempfert, H.-G (1999), Alexiew, D., Brokemper, D. and Lothspeich, S. (2005), Alexiew, D., Raithel, M. (2015) for GEC, which is formed on the unit cell concept where...
the column is considered as rigid plastic, with infinite modulus of elasticity.

A geotextile encased stone column (GEC) in soft soil under a maintained load is usually subject to additional settlement, the magnitude of which differs because of changes in the stress-strain behavior over time. Such stress relaxation behavior occurs in the soil surrounding the GEC as well as on the GEC-soil interface itself. This article focuses only on the generalized Maxwell model of viscoelastic soft soil and Relaxation Modulus $E(t)$. The Maxwell model, which consists of spring and dashpot element in series, can effectively reflect the stress relaxation behavior of soft soil.

**Estimating the viscosity and strain rate**

The viscosity and strain rate of the soft clay will be taken from the studies of Anders Augustesen, Morten Liingaard and Poul V. Lade (2004)\(^6\), G. Qu, S. D. Hinchberger and K. Y. Lo (2010)\(^7\), Arindam Dey and Prabir Kr. Basudhar (2012)\(^8\), Hong-Hu Zhu, Lin-Chao Liu, Hua-Fu (2012)\(^9\).

**Varying the relaxation times and elasticity modulus**

The relaxation time is dependent on the value of young's modulus $E$ and the viscosities $\eta$. In this paper $\alpha = 0.5$ will assumed for all the analysis\(^4\).

\[
\tau = \frac{\alpha \cdot \eta}{1 - \alpha} \cdot E
\]  

**Settlement calculation of GEC and the time-dependent behavior of soft clay soils**

The ground improvement works are to provide a steady platform to support the operation of SL6000 (Kobelco) and the CC2800 (Terex Demag) cranes on the designated routes, which can support up to 500 kPa of transient loads and long-term primary settlement should be less than 250 mm. The soft clay is reinforced by the GEC with a diameter of 0.8 m, depth of treatment from 12 m and the tensile stiffness of the geotextile encasement $J = 3000$ kN/m. The GEC are arranged in square grids with spacing 2.3 m. The settlement was estimated based on Raithel and Kempfert’s analytical calculation\(^10\). This calculation is conducted using data obtained from soil parameters in Table 1.

**ANALYSIS, RESULTS AND DISCUSSION**

A Matlab code was written to simulate viscoelastic behavior of soft clay. The model was built first only with the spring, then a dashpot was added to the model to simulate the mechanics of a Maxwell element. The generalized Maxwell model was simulated by combining a spring and five Maxwell elements in this paper. For the present study, the design variables governing the constitutive behavior of the generalized Maxwell model for soft soil are as follows and listed in Table 2: (1) Elastic coefficient of the Maxwell element $E_{spring}$, (2) Viscous coefficient of the Maxwell element $\eta_i$, (3) Strain rate $\dot{\varepsilon}$.

To study the suitability of the generalized Maxwell model, the viscosity $\eta$ is set to 0.1, 0.5, 1, 5 and 10 MPa·d and strain rate $\dot{\varepsilon}$ is checked with 1%, 5%, 10% and 20%.

How to determine the Maxwell model parameters is a major concern, in this section some issues are considered to evaluate the response of the generalized Maxwell model described above. Several studies are performed and relaxation time is considered, which is usually used to assess the time-dependent behavior of viscoelastic soft soil.

**Constant strain rate with different viscosities**

In this case the influence of viscosity on the generalized Maxwell model is studied. The numerical simulations are carried out with different number of Maxwell elements. It can be seen that when the viscosity or the number of Maxwell elements increases greater values of stress are
Table 1: Typical Subsoil Profile

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil Type</th>
<th>$\phi_{orig}$</th>
<th>$c'_{orig}$ (kPa)</th>
<th>Elastic Modulus E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.6</td>
<td>Gravel</td>
<td>40</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>0.6 to 1.5</td>
<td>Sand</td>
<td>30</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1.5 to 2.5</td>
<td>Crust</td>
<td>0</td>
<td>35</td>
<td>-</td>
</tr>
<tr>
<td>2.5 to 13.5</td>
<td>Soft Clay</td>
<td>0</td>
<td>20</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2: Parameters of the generalized Maxwell model

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus E</td>
<td>1.0</td>
<td>MPa</td>
</tr>
<tr>
<td>Viscosity $\eta$</td>
<td>varies</td>
<td>MPa-d</td>
</tr>
<tr>
<td>Strain rate $\dot{\varepsilon}$</td>
<td>varies</td>
<td>-</td>
</tr>
<tr>
<td>Linear elastic Modulus $E_{spring}$</td>
<td>0.2</td>
<td>MPa</td>
</tr>
</tbody>
</table>

obtained. This is highlighted in Figure 5, in which the effect of numbers of Maxwell elements on the maximum stress $\sigma$ reached can be observed.

The general observation is that the higher the viscosity, the higher the stresses for a certain strain rate. For example, Figure 5 (a and b) show the comparison of the stress versus time by viscosity 5 Mpa·d and 10 Mpa·d with same constant strain rate 5% per day for a period of 500 days, and it is observed that the stress reached a maximum level 1,428 MPa and 2,365 MPa respectively after a period of time of 20 days for case 5 Maxwell elements.

The evolution of stress with time for different number of Maxwell elements is shown in Figure 5. As can be seen the stress response is nonlinear although the strain rate has a linear variation with time.

**Constant Maxwell elements with different strain rates and viscosities**

In this case two different viscosities with five different strain rates are considered. From a point of view of the sensitivity to constitutive parameters it can be said that when viscosity decreases, the stress is lower for the same time.

It is seen that the maximum stress moves to the right faster for higher strain rate. In Figure 6, the results of the generalized Maxwell model are shown, and it seems that the bigger the strain rate, the stiffer the soil. Figure 6 also shows the results obtained by changing strain rate from 1% to 20% while keeping the other parameters constant. It can be seen that the smaller the value of strain rate, the longer the time to reach the maximum stress. An example, Figure 6a shows the result of strain rate 1% per day, during the first 100 days an increasing strain is applied, the strain rate is constant and equals 1% per day, during these days the stress appears to increase non-linearly with the strain. After 100 days the strain remains constant at maximum strain, and the stress decreases exponentially to a stress-relaxation limit.

Figure 7 presents the effects of strain rate on the stress of the generalized Maxwell model for soft soil. For the short period of time (1 to 5 days), the stress is found to increase linearly with strain rate before the required strain value is reached. The stress developed almost linearly with low strain rate (1% to 10% per day) until the maximum strain is reached and then decreases with high strain rate for a period of 10 to 20 days. Finally, for the larger period of more than 50 days, the stress is gradually reduced from very low strain rate to very high strain rate.

**Settlement calculation of GEC**

The settlement of a GEC is estimated using the elastic modulus $E$ and relaxation modulus $E(t)$. It is shown that the viscosity mainly affects the overall relaxing rate of the foundation soil. For the same time and same strain rate, the settlements increase with the decrease of viscosity.

The increase of Maxwell elements causes smaller values of settlement, when the viscosity and strain rate are same. Fig. 8 shows the results at different moduli of relaxation modulus $E(t)$ under constant loading with the viscosity $\eta = 5$ Mpa·d and 10 Mpa·d.

It is easy to make the conclusion that the smaller the modulus of relaxation $E(t)$ is, the bigger the settlement is, and the smaller the strain rate to reach the
Figure 5: Stress versus time for constant strain rate
final settlement spends.

**LIMITATION OF THE PROPOSED APPROACH**

The proposed analysis of using relaxation modulus for estimating the settlement of GEC is primarily intended for understanding the behavior of viscoelastic soils. It has been mentioned in the study that the generalized Maxwell model is capable of representing the behavior of a viscoelastic soil when considered in terms of the short-term or long-term settlement under loading followed by a relaxation phase under sustained loading. The used data surely restricts the approach of estimating the generalized Maxwell model parameters being valid only for the loaded viscoelastic soft soils.

The verifiable relationship between the generalized Maxwell model parameters and soft soil properties is an important problem to be solved, which needs the support of sufficient measurement data from laboratory and field experiments.

**CONCLUSIONS**

Based on the above studies, the following conclusions can be made:

- Based on the theory of linear viscoelasticity, a generalized Maxwell viscoelastic model is developed to account for the time-dependent behavior of soil foundations improvement with GEC under concentrated line load.
- Analytical solution of settlement in the foundation was estimated based on Raithel and Kempfert’s analytical calculation model with Relaxation Modulus.
- The viscoelastic theory shows its potential in modeling the long-term foundation deformation.
The presented analytical calculation can be extended to solve other geotechnical problems.

CONFLICT OF INTEREST
The authors pledge that there are no conflicts of interest in the publication of the paper.

AUTHORS’ CONTRIBUTION
Pham Tien Bach presented the idea of study and carried out the collecting data, writing codes and writing the paper manuscripts. Dr. Vo Dai Nhat, Assoc. Prof. Dr. Nguyen Viet Ky participated in the scientific idea of research, guided to writing the paper, reviewed the results of study. Le Quan contributed to review the input data, output data and reviewing the paper.

APPENDIX: MATLAB CODE

```
dt=0.1;
t=0:dt:500;
%Boundary conditions
strain(1) = 0;
straindot(1) = 0;
stress1(1) = 0;
stressdot(1) = 0;
prompt={'Enter the maximum strain','Enter the strain rate','Enter Viscosity Eta1','Enter Viscosity Eta 2','Enter Viscosity Eta 3','Enter Viscosity Eta 4','Enter Viscosity Eta 5','Enter Youngs Modulus','Enter Linear Modulus'};
name='Sample setup';
umlines=1;
defaultvalue={'1','0.2','10','10','10','10','10','1.0','0.2'};
answer=inputdlg(prompt,name,numlines,defaultvalue);
maxstrain = str2num(answer{1});
strainrate = str2num(answer{2});
visco1 = str2num(answer{3});
visco2 = str2num(answer{4});
```
Figure 8: Settlements calculated using elastic and generalized Maxwell model

a. $\varepsilon = 1\%$ and $\eta = 5$ MPa-d & 10 MPa-d

b. $\varepsilon = 5\%$ and $\eta = 5$ MPa-d & 10 MPa-d

c. $\varepsilon = 10\%$ and $\eta = 5$ MPa-d & 10 MPa-d

d. $\varepsilon = 10\%$ and $\eta = 5$ MPa-d & 10 MPa-d
visco3 = str2num(answer{5});
visco4 = str2num(answer{6});
visco5 = str2num(answer{7});
E = str2num(answer{8});
Espring = str2num(answer{9});
strain = zeros(size(t));
stress = zeros(size(t));
stress1 = zeros(size(t));
stress2 = zeros(size(t));
stress3 = zeros(size(t));
stress4 = zeros(size(t));
stress5 = zeros(size(t));
stressspring = zeros(size(t));
stressdot1 = zeros(size(t));
stressdot2 = zeros(size(t));
stressdot3 = zeros(size(t));
stressdot4 = zeros(size(t));
stressdot5 = zeros(size(t));
straindot = zeros(size(t));
alpha = 0.5;
relaxationtime1 = (alpha*visco1)/((1-alpha)*E);
relaxationtime2 = (alpha*visco2)/((1-alpha)*E);
relaxationtime3 = (alpha*visco3)/((1-alpha)*E);
relaxationtime4 = (alpha*visco4)/((1-alpha)*E);
relaxationtime5 = (alpha*visco5)/((1-alpha)*E);
for i = 2:length(t)
    strain(i) = strain(i-1)+ strainrate * dt;
    if strain(i) > maxstrain
        strain(i) = maxstrain;
    end
    straindot(i) = (strain(i)-strain(i-1))/dt;
    if strain(i) > maxstrain
        strain(i) = maxstrain;
    end
relaxationtime1 <= 0
    stressdot1(i) = 0;
stress1(i) = 0;
else
    stressdot1(i) = (- stress1(i-1) + visco1* straindot(i) ) / relaxationtime1;
    stress1(i) = stress1(i-1) + stressdot1(i) * dt;
    if relaxationtime1 <= 0
        stressdot1(i) = 0;
    end
    if relaxationtime2 <= 0
        stressdot2(i) = 0;
    else
        stressdot2(i) = (- stress2(i-1) + visco2* straindot(i) ) / relaxationtime2;
        stress2(i) = stress2(i-1) + stressdot2(i) * dt;
    end
    if relaxationtime3 <= 0
        stressdot3(i) = 0;
    else
        stressdot3(i) = (- stress3(i-1) + visco3* straindot(i) ) / relaxationtime3;
        stress3(i) = stress3(i-1) + stressdot3(i) * dt;
    end
    if relaxationtime4 <= 0
        stressdot4(i) = 0;
    else
        stressdot4(i) = (- stress4(i-1) + visco4* straindot(i) ) / relaxationtime4;
        stress4(i) = stress4(i-1) + stressdot4(i) * dt;
    end
    if relaxationtime5 <= 0
        stressdot5(i) = 0;
        stress5(i) = 0;
    else
        stressdot5(i) = (- stress5(i-1) + visco5* straindot(i) ) / relaxationtime5;
        stress5(i) = stress5(i-1) + stressdot5(i) * dt;
    end
    stressspring(i) = Espring * strain(i);
    stress(i) = stress1(i) + stress2(i) + stress3(i) + stress4(i) + stress5(i)+ stressspring(i);
end

%Plotting
subplot(211); plot(t,stress, 'black' , 'LineWidth' ,1); xlabel('time (days)' , 'FontSize' ,12); ylabel('stress (MPa)' , 'FontSize' ,12);
subplot(212); plot(t,strain, 'black' , 'LineWidth' ,1); xlabel('time (days)' , 'FontSize' ,12); ylabel('strain' , 'FontSize' ,12);

REFERENCES


Mô hình Maxwell cho nền đất yếu gia cố bằng cọc cát/đá bọc vải địa kỹ thuật

Phạm Tiến Bách*, Võ Đại Nhật, Lê Quân, Nguyễn Việt Kỳ

TÓM TẮT
Trong lĩnh vực địa kỹ thuật – cải tạo nền đất yếu, các mô hình toán hay cơ là một trong những thông số đầu vào rất quan trọng phục vụ thiết kế tính toán hay nghiên cứu. Việc xác định sự phù hợp của các mô hình có ảnh hưởng rất lớn đến độ chính xác kết quả thiết kế và tính toán cũng như tính ổn định bền vững của nền đất yếu sau khi được cải tạo. Ngược lại, việc lựa chọn các mô hình tính toán không phù hợp sẽ dẫn đến chi phí cải tạo nền đất yếu tăng cao, thậm chí có thể dẫn đến việc mất ổn định công trình và gây ra các thiệt hại lớn về kinh tế.

Gần đây, rất nhiều dự án đường cao tốc sau khi thiết kế thì công đưa vào sử dụng thì không đáp ứng yêu cầu của tiêu chuẩn dẫn đến hao tổn kinh phí của các cá nhân, tổ chức và nhà nước để xử lý hậu quả. Do đó việc nghiên cứu và ứng dụng sử dụng các mô hình toán hay cơ phù hợp với phương pháp cải tạo nền đất yếu mới sẽ giúp ích rất nhiều cũng như bổ sung thêm các lựa chọn cho công tác cải tạo đất yếu tại Việt Nam. Biến dạng của nền đất yếu không chỉ liên quan đến tải trọng mà còn liên quan đến thời gian giao tải. Sự thay đổi ứng suất và biến dạng của nền đất yếu theo thời gian được gọi là đặc tính lưu biến, trong nghiên cứu này là ứng suất của tính đàn – nhớt. Từ những lý do trên, chúng tôi đã giao áp dụng mô hình tổng quát Maxwell để giải thích ứng suất có tính đàn – nhớt của nền đất yếu. Đặc biệt, ứng dụng phân tích mô hình tổng quát Maxwell giúp giải quyết biến dạng sét cát các phương trình toán học về các kết quả mô hình tổng quát Maxwell. Chúng tôi thử nghiệm những mô hình tổng quát Maxwell có thể sử dụng trong việc nghiên cứu ứng suất phụ thuộc theo thời gian của nền đất yếu. Kết quả cho thấy mô hình này là một trong những mô hình hiệu quả để dự đoán ứng suất của nền đất yếu trong việc cải tạo nền đất yếu với cọc cát/dá bọc vải địa kỹ thuật.

Từ khóa: Mô hình Maxwell, cọc cát/dá bọc vải địa kỹ thuật, cải tạo nền đất yếu, ứng suất đàn – nhớt