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Research Article

A general mathematical model to determine temperature and moisture distributions in drying of spherical products: Case studies of green peas, barley, and cranberry

Pham Ba Thao[1,2], Duong Cong Truyen[2], Nguyen Minh Phu[1,*]

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ABSTRACT

Forced convection drying is a simple and effective dehydration method widely applied in post-harvest technology. Sphere-shaped materials account for a large proportion compared to other shapes. Therefore, this study forms a general mathematical model and resolution method to predict the temperature and moisture content of spherical drying materials with time. Three representative spheres of green peas, cranberries, and barley were used as case studies to validate the simulation results with experimental results. All three crops are widely used in cooking. Green peas are often used in stews, salads or processed into soybeans. Cranberries are frequently employed in desserts, juices or processed into jam. Barley is often used to make bread and beer and is also an important grain source in human nutrition. The Arrhenius model predicts the moisture diffusion coefficient with temperature to link the temperature and moisture equations, and they are solved simultaneously. In drying applications, the Arrhenius model is often utilized to describe the temperature dependence of moisture diffusion coefficients or drying rates. By applying the Arrhenius equation to drying kinetics, the relationship between drying rate and temperature can be quantified, allowing for the prediction and optimization of drying processes under different temperature conditions. These three objects have very different input parameters to illustrate the wide applicability of the proposed solution method. The results show that there is a large difference in moisture content at the center and surface of a crop. On the contrary, the temperature is evenly distributed, and the dried object quickly reaches the drying air temperature. The laws of heat transfer and moisture transfer are analogy. However, the moisture diffusion coefficient and moisture transfer coefficient are much smaller than the thermal diffusion coefficient and heat transfer coefficient.

Key words: Heat-mass transfer coupling, hot air drying, Arrhenius equation, agricultural product, numerical method

[1]*Faculty of Heat and Refrigeration Engineering, Industrial University of Ho Chi Minh City (IUH), Ho Chi Minh City 700000, Vietnam*

[2]*Faculty of Mechanical Engineering, Industrial University of Ho Chi Minh City (IUH), Ho Chi Minh City 700000, Vietnam*

Correspondence

Nguyen Minh Phu, Faculty of Heat and Refrigeration Engineering, Industrial University of Ho Chi Minh City (IUH), Ho Chi Minh City 700000, Vietnam

Email: nguyenminhphu@iuh.edu.vn

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INTRODUCTION

Crop drying plays a crucial role in agricultural production and post-harvest management. Its primary objectives are to reduce the moisture content of harvested crops to safe levels, preserve their quality, and prevent spoilage[1]. Studying the kinetics of drying process is a necessary preliminary step to determine the drying model and parameters such as drying constant, moisture transfer coefficient, moisture diffusion coefficient, and heat transfer coefficient. From these parameters, engineer can use to predict moisture distribution and temperature inside the drying object[2]. This prediction can be done in three ways: Analytical solution, numerical simulation, or self-developed code. The analytical method is quite complicated because it involves partial differential equations (PDEs). Numerical simulation methods require high-configuration computers and simulation software, e.g., ANSYS, Comsol. The self-developed code method can use popular and compact software such as Excel, EES, Matlab to compose calculation programs for objects with simple geometries such as rectangles, cylinders and spheres[3]. Most fruits, vegetables, and seeds are spherical in shape, such as green peas, barley, cranberries, grapes, and plums[4–6]. In this study, the first three types of agricultural products were selected as case studies.

Green peas, scientifically known as Pisum sativum, are a versatile and nutritious vegetable widely cultivated and consumed around the world. Belonging to the legume family, green peas are characterized by their small, round shape and vibrant green color. They are commonly consumed fresh, frozen, or dried and are renowned for their sweet flavor and tender texture[7]. Cranberries are widely used in the production of juices, sauces, jams, and dried fruit snacks. Cranberry juice is particularly popular and is often consumed for its potential health benefits, including supporting urinary tract health[8]. Barley is a

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good source of vitamins and minerals, including vitamin B6, niacin, thiamine, riboflavin, iron, magnesium, phosphorus, and selenium. It also contains antioxidants such as phenolic acids, flavonoids, and tocopherols, which help protect against oxidative stress and inflammation[9].

It is difficult to measure the temperature and moisture inside the drying object because of its small size. But the temperature or moisture is important since it is possible that the surface moisture is as desired but not at the center, affecting the storage time and lifespan. Therefore, this study is to develop a general numerical model that can predict the temperature and moisture distribution inside a spherical agricultural product. The governing equations, numerical method, parameters required for simulation, and validation of simulation results are presented and analyzed.

HEAT-MASS TRANSFER COUPLING MODEL AND NUMERICAL METHOD

Let the initial spherical moist object have radius r_0, moisture content M_0 and temperature T_0 as shown in Figure 1. Distribution of temperature (T) and moisture (M) of the object according to radius (r) and time (t), i.e., T(t,r) and M(t,r), respectively can be described by the heat conduction equation and Fick's diffusion equation as follows:

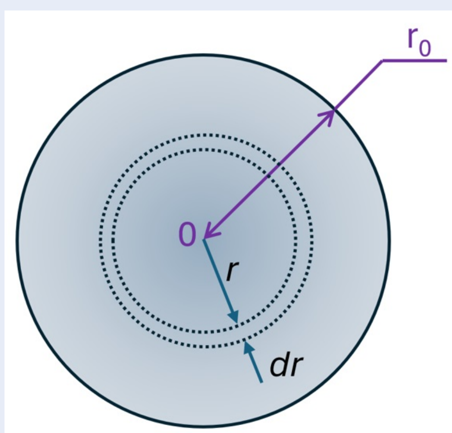

Figure 1: Coordinate along sphere radius

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{1}$$

$$\frac{1}{D}\frac{\partial M}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial M}{\partial r}\right) \tag{2}$$

where α is thermal diffusivity (m^2/s),
D is moisture diffusivity (m^2/s),
M is dry basis moisture content (kg/kg db),
r is space coordinate (m),
T is temperature (°C),
t is time (s).

Boundary conditions for Eqs. (1) and (2) are as follows:

$$\frac{\partial T}{\partial r}|_{r=0} = 0, \tag{3}$$

$$\frac{\partial M}{\partial r}|_{r=0} = 0, \tag{4}$$

$$-k\frac{\partial T}{\partial r}|_{r=r_0} = h\left(T - T_d\right), \tag{5}$$

$$-D\frac{\partial M}{\partial r}|_{r=r_0} = h_m\left(M - M_e\right), \tag{6}$$

where T_d is drying temperature (°C),
k is thermal conductivity of the product (W/m K),
h is the heat transfer coefficient (W/m^2 K),
h_m is mass transfer coefficient (m/s),
M_e is equilibrium moisture content (kg/kg db).

The system of differential equations above and the associated boundary conditions can be solved using the finite difference method. With the spatial discretization as shown in Figure 2, the implicit method and central difference scheme are applied to Eqs. (1) and (2) into discretization equations as follows:

$$\frac{1}{\alpha}\frac{T_i^{n+1} - T_i^n}{\triangle t} = \frac{T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}}{\triangle r^2} + \frac{2}{r}\frac{T_{i+1}^{n+1} - T_{i-1}^{n+1}}{2\triangle r} \tag{7}$$

$$\frac{1}{D}\frac{M_i^{n+1} - M_i^n}{\triangle t} = \frac{M_{i+1}^{n+1} - 2M_i^{n+1} + M_{i-1}^{n+1}}{\triangle r^2} + \frac{2}{r}\frac{M_{i+1}^{n+1} - M_{i-1}^{n+1}}{2\triangle r} \tag{8}$$

Discrete equations of boundary conditions are given as:

$$T_1 = T_2 \tag{9}$$

$$M_1 = M_2 \tag{10}$$

$$T_N = \frac{h\triangle r T_d + kT_{N-1}}{k + h\triangle r} \tag{11}$$

$$M_N = \frac{h_m\triangle r M_e + DM_{N-1}}{D + h_m\triangle r} \tag{12}$$

The discrete equations (7) and (8) form systems of three-diagonal linear equations and are solved using the Thomas algorithm.

Figure 2: Spatial discretization

RESULTS AND DISCUSSION

This section illustrates the results using the finite difference method mentioned above. Spherical moist objects consisting of green peas, cranberries, and barley were used as case studies for application of the mathematical model and resolution method. The parameters of the spheres are shown in Table 1. The number of nodes is set N = 30 in the computational method. This number was selected after a grid independence test. The self-developed code was implemented in MATLAB software. Moisture diffusion coefficient (D) according to drying temperature (T in Kelvin) is calculated from the Arrhenius equation. The equation has the following form [10]:

$$D = D_0 exp\left(-\frac{E_a}{R_g T}\right) \tag{13}$$

where D_0 is pre-exponential factor (m^2/s),
E_a is activation energy (J/mol),
R_g is universal gas constant, R_g = 8314 J/mol K.

Figure 3 shows the moisture content profiles of three spherical objects compared to experimental data. The unit of x-axis was different from Figure 3a, b, and c because the three test-cases are different materials. There is good agreement between the simulation results in this study and the experimental results, especially with drying green peas. For cranberries at about 4-hour drying time and Barley drying after 150 s, the moisture content prediction of the present study is slightly greater than the experimental data. This can be explained by the fact that the current mathematical model does not consider shrinkage of the drying material. When shrinkage is taken into account, the moisture content of the calculation will be reduced.

The internal moisture content of the objects with time from the simulation is seen in Figure 4. The scale of x-axis was different between subfigures because the three test-cases have different diameters. The diameter of cranberry is much more larger those of green peas and barley. After one hour of drying, the moisture content at the center of the green peas decreased from 3 to 2.6 kg/kg db. There is a huge moisture difference of up to 2 kg/kg db at the surface of the peas and the center at this time. Because this is the stage of surface moisture evaporation. After 5 hours of drying, the temperature inside the peas is relatively uniform. Similar developments occurred for cranberries. As for barley, due to its very small size and small initial moisture content, the drying time is quite short. In the first 50 s, there is only a change in moisture close to the barley's surface.

Figure 5 presents the temperature variation inside the three spheres. Due to the small size of the objects, after a short drying time, the temperature of the products approaches the drying temperature. In addition, the thermal diffusivity is larger than the moisture diffusivity, so the temperature distribution is relatively more uniform than the moisture distribution at the same time. This means that the heat transfer Biot number (= $2r_0h/k$) is less than the mass transfer Biot number (= $2r_0h_m/D$).

CONCLUSIONS

The partial differential equations of heat transfer and moisture transfer describing drying of sphere-shaped material are presented in this article. The finite difference method is proposed to discretize the equations and boundary conditions. Three spherical objects were reviewed for the necessary drying parameters, and the drying curve was compared between the numerical approximation and experiment. The results show great agreement with experimental data. Therefore, the current mathematical model can be applied to many different spherical drying objects to determine the temperature and moisture distribution inside the object as well as the average temperature and moisture with drying time. In addition, water evaporation and shrinkage should be expanded into consideration to improve the mathematical model and reduce deviations from experimental results.

CONFLICT OF INTEREST

There is no conflict of interest.

AUTHORS' CONTRIBUTION

Pham Ba Thao: hardware, software, validation, formal analysis, writing draft.
Duong Cong Truyen: derivation, program.
Nguyen Minh Phu: review and editing.

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Table 1: Necessary inputs to the mathematical modeling

No.	Parameter	Green peas[11–13]	Cranberry[4,14]	Barley[10,15]
1	r_0, m	0.005	0.01	0.002
2	h, W/m^2 K	3.65	8.55	110
3	T_d, °C	60	90	75
4	M_e, kg/kg db	0.196	0.196	0.0725
5	M_0, kg/kg db	3	7.492	0.27
6	T_0, °C	25	27	33
7	k, W/m K	0.5	0.248	0.3
8	a, m^2/s	$1.54x10^{-7}$	$0.106x10^{-6}$	$1.8116x10^{-7}$
9	hm, m/s	$1x10^{-6}$	0.01	0.0069
10	D, m^2/s	$1.82x10^{-8}$exp(-1119/T)	$3.32x10^{-4}$exp(-4431.69/T)	$1.56x10^{-5}$exp(-3086/T)

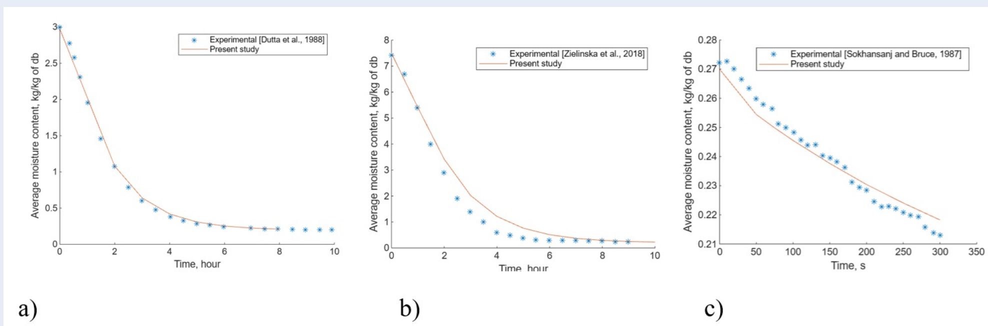

Figure 3: Verification of moisture content with experimental data. a) Green peas [16], b) Cranberry [17], c) Barley [18]

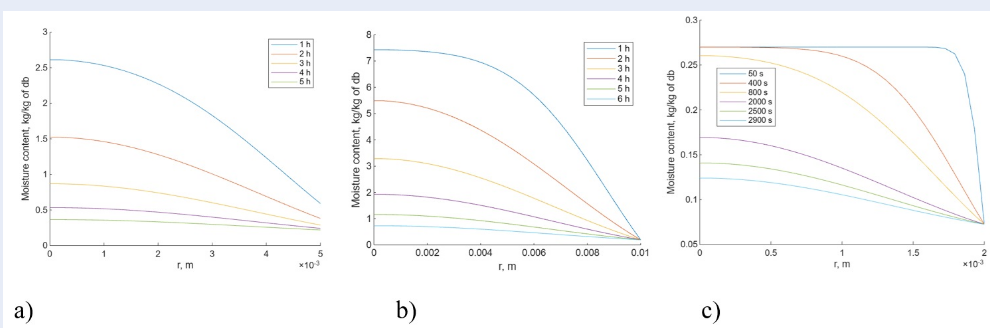

Figure 4: Moisture content distribution with time. a) Green peas, b) Cranberry, c) Barley

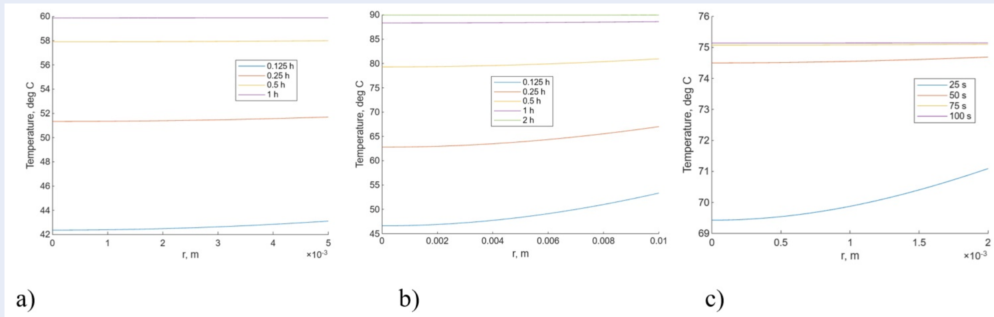

Figure 5: Temperature of spherical products with time. a) Green peas, b) Cranberry, c) Barley

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Bài nghiên cứu

Mô hình toán chung xác định phân bố nhiệt độ và độ ẩm khi sấy vật hình cầu: đậu hà lan, lúa mạch, và nam việt quất

Phạm Bá Thảo[1,2], Đường Công Truyền[2], Nguyễn Minh Phú[1,*]

TÓM TẮT
Sấy đối lưu cưỡng bức là phương pháp sấy đơn giản và hiệu quả được áp dụng rộng rãi trong công nghệ sau thu hoạch. Vật sấy hình cầu chiếm một tỷ lệ lớn so với các hình dạng khác. Do đó nghiên cứu này thành lập mô hình toán và lời giải chung để dự đoán nhiệt độ và độ ẩm của vật sấy hình cầu theo thời gian. Ba vật cầu tiêu biểu gồm đậu Hà Lan, nam việt quất, và lúa mạch được sử dụng như các nghiên cứu điển hình để xác nhận kết quả mô phỏng so với thực nghiệm. Tất cả ba loại quả này đều được sử dụng rộng rãi trong nấu ăn. Đậu Hà Lan thường được sử dụng trong các món hầm, salad hoặc chế biến thành đậu nành. Quả nam việt quất thường được sử dụng trong các món tráng miệng, nước ép hoặc chế biến thành mứt. Lúa mạch thường được sử dụng để làm bánh mì, bia, và cũng là một nguồn ngũ cốc quan trọng trong dinh dưỡng con người. Mô hình Arrhenius dự đoán hệ số khuếch tán ẩm theo nhiệt độ để liên kết các phương trình nhiệt độ và độ ẩm và các phương trình được giải đồng thời. Ba vật này có các thông số nhập vào rất khác nhau để minh họa khả năng ứng dụng rộng của phương pháp giải được đề xuất. Kết quả cho thấy có sự chênh lệch lớn độ ẩm tại tâm và bề mặt của vật sấy. Ngược lại nhiệt độ phân bố đều và vật sấy mau đạt đến nhiệt độ không khí sấy. Quy luật truyền nhiệt và truyền ẩm là tương tự. Tuy nhiên hệ số khuếch tán ẩm và hệ số truyền ẩm là rất nhỏ hơn hệ số khuếch tán nhiệt và hệ số truyền nhiệt.

Từ khoá: Truyền nhiệt và truyền khối, sấy không khí nóng, phương trình Arrhenius, nông sản, phương pháp số

[1]*Khoa công nghệ nhiệt lạnh, Trường Đại học Công nghiệp Thành phố Hồ Chí Minh (IUH), Việt Nam*

[2]*Khoa công nghệ cơ khí, Trường Đại học Công nghiệp Thành phố Hồ Chí Minh (IUH), Việt Nam*

Liên hệ

Nguyễn Minh Phú, Khoa công nghệ nhiệt lạnh, Trường Đại học Công nghiệp Thành phố Hồ Chí Minh (IUH), Việt Nam

Email: nguyenminhphu@iuh.edu.vn

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