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Research Article

Trajectory planner based on fifth-order polynomials applied for lane changing in autonomous driving

Duc Lich Luu*, Thanh-Long Phan, Tien Thua Nguyen, Vinh Quang Huynh, Dac Thanh Dat Nguyen

ABSTRACT

This paper focuses on the motion control of autonomous cars, particularly for executing safe lane change maneuvers. The proposed solution integrates Model Predictive Control (MPC) with a fifth-order polynomial trajectory planner to handle lane changes and avoid collisions in dynamic driving environments. The primary advantage of this approach is its minimal computational resource requirements, making it suitable for real-time deployment while maintaining high performance in complex traffic conditions. The paper starts by developing a nonlinear dynamic model of the car, emphasizing lateral dynamics, which is crucial for planning and controlling the car's movement during lane changes. The model accounts for important parameters like yaw rate, lateral forces, and steering angles. The trajectory planner is designed to calculate an optimal, collision-free path for the car to follow when changing lanes or overtaking other cars, ensuring that the car stays within safety constraints, such as maintaining an appropriate distance from preceding cars. A novel aspect of the proposed solution is the integration of decision-making with trajectory planning. The system calculates the safe distance from the preceding car using time-to-collision and inter-vehicular time metrics. These metrics enable the car to decide whether to stay in its lane or initiate a lane change, based on the safety of the maneuver. Once a decision is made, the trajectory planner generates a new reference path, ensuring a safe and smooth lane change, even in the presence of obstacles. The effectiveness of the proposed system is demonstrated through extensive simulations in a variety of driving scenarios. These simulations show that the car can successfully perform lane changes and overtakes without colliding with other cars, while maintaining comfort and minimizing control errors. The simulation results validate that the MPC-based control system, combined with the polynomial trajectory planner, offers a reliable and efficient solution for real-time trajectory planning and control in autonomous driving.

Key words: MPC, trajectory planner, obstacle avoidance, decision making, car dynamics, car control

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INTRODUCTION

The advancement of car technology, coupled with the implementation of various driver assistance systems, has played a significant role in the development of smart cars capable of autonomous driving [1–3]. The deployment of autonomous cars on highways can yield numerous benefits, with a notable one being the decrease in traffic accidents.

Accordingly, the trajectory planner and trajectory tracking [4] aim to formulate a control input vector for an autonomous driving based on a predefined objective function. This enables the autonomous car to follow the desired trajectory at a predetermined velocity. Its core objective is to minimize both time and spatial discrepancies between the car and the reference path through precise control of car motion. How to make an effective control aprroach to guarantee the control accuracy [5], and address real-time response demands [6], path tracking control has emerged as a research focal point within the field, garnering significant attention. Currently, among the widely adopted path following strategies, the pure pursuit algorithm stands out as a prominent choice [7], PID control algorithm [8] and fuzzy control algorithm [9].

Recently, methods based on MPC, often referred to as receding horizon control, are commonly employed for controlling dynamic systems, particularly in the area of autonomous cars. This algorithm is particularly effective in intuitively handling multi-variable constrained control challenges. Numerous studies have applied Model Predictive Control (MPC) to address overtaking and obstacle avoidance challenges, summarized as follows: an algorithm utilizing sensor data logic selects the optimal evasive maneuver for the car to bypass obstacles [10]. The authors implement a random tree strategy to create a trajectory planner aimed at circumventing stationary obstacles [11]. The authors introduces a random tree star approach to design a collision-free path, specifically for exploration

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scenarios[12]. Additionally, research into obstacle detection and emergency avoidance has continued to advance: the MPC-based collision avoidance system maintains fixed safety distances[13], with time-variant behavior introduced only when the ramp barrier's upper bound is extended to infinity, effectively deactivating it. A comparable approach is presented[14], utilizing reachable sets to anticipate the behavior of surrounding cars and guarantee the feasibility of the planned collision-free trajectory. An MPC controller uses path velocity decomposition to support collision avoidance for autonomous driving[15], integrating a time-scaled collision cone and formulating the forward speed optimization as a convex quadratic programming problem. Finally, a model-based MPC method is proposed for emergency obstacle avoidance applications[16,17], while cloud clustering is used to enhance obstacle detection[18]. Furthermore, the MPC was utilized to assess the likelihood of conflicts during the overtaking maneuver[19]. Finally, a collision avoidance system was suggested[20], employing a combination of deep learning and MPC to execute car maneuvers like overtaking. Certainly, MPC provides a direct method for incorporating environmental constraints (such as road limits and navigable zones) as well as car dynamic constraints (such as actuator limits, etc.) into the optimization problem. The decision to utilize MPC for collision avoidance in this paper is driven by the advantages mentioned above. Despite the simulation results demonstrating the desirable performance in these researches, It is worth to note that the researches are lacking a new solution for trajectory planning tool design, the inspiration starting from the researches[21,22]. The proposed solution in this paper utilizes information regarding the distance to the obstacle and calculates the timing for lane changes necessary for the car.

The distinctiveness of our solution compared to studies in references can be outlined as follows: i) The proposed solution requires minimal computational resources, enabling practical real-time deployment; ii) Integration of decision-making with trajectory planning: This approach segments the host car's space by calculating a safe distance from the previous car, using metrics like time to collision and inter-vehicular time. Based on these safety metrics, lane change decisions are made, and a trajectory planner is engaged to avoid collisions. The planner then creates a new reference path to guide the car safely around obstacles.

The structure of the remaining sections is organized as follows: Section 2 details the car modeling for the car, while Section 3 provides a problem description, encompassing trajectory planning integrated with physical and collision avoidance constraints. Section 4 outlines the control unit design, and illustrative results are discussed in Section 5. Lastly, concluding remarks are presented in Section 6.

CAR MODELING

The car is assumed to be driving without considering road excitation that may induce vertical motion, pitch, and roll. The car dynamics model considers only three degrees of freedom—longitudinal, lateral, and yaw—as illustrated in Figure 1. The full 4-wheel model is mentioned[23], the simulation results give the velocity for each wheel, however the aim of the paper is only to consider the kinematics including velocity, acceleration at the center of gravity of the car. Therefore, to simplify controller design, less complex yet tractable models are employed. The right and left wheels for each axle are collapsed into a single representation, call is a bicycle model, as illustrated in Figure 1. Under certain assumptions, the bicycle model adequately captures the essential lateral dynamics required for controller synthesis. These assumptions are as follows: The car is symmetric about the longitudinal plane, meaning the left and right sides are identical. The tire behavior is assumed to be linear, and the wheels do not experience slipping; The analysis disregards the vertical movement of the car and the impact of the suspension, focusing solely on the lateral and yaw motions.

Car longitudinal movement for the bicycle model:

$m(a_x - v_y.\psi) = F_{x_t} + F_{x_s} - F_r$ (1)

Lateral movement:

$m(a_y - v_x.\psi) = F_{y_t} + F_{y_s}$ (2)

Yaw movement:

$m(a_y - v_x.\psi) = F_{y_t} + F_{y_s}$ (3)

where m- mass of the car. $v_x a_x$ - speed and acceleration along the longitudinal direction of the car, respectively. $v_y a_y$speed and acceleration along the horizontal direction of the car. ψ, ω- yaw angle and yaw rate of the car. F_{y_t} , F_{y_s}- longitudinal and lateral forces acting on the front and rear axles of a car. a, b - distance from the car's center of gravity to the front and rear wheels. I_z- rotational inertia of the car about the vertical axis.

The car's motion with respect to global coordinates may be expressed as

$$\begin{cases} v_x = [\cos\psi \; - \; \sin\psi \begin{bmatrix} v_x \\ v_y \end{bmatrix} \\ v_y = [\sin\psi \quad \cos\psi \begin{bmatrix} v_x \\ v_y \end{bmatrix} \end{cases} \quad (4)$$

where V_Y is the horizontal speed of the car in the inertial coordinate system, v_X is the longitudinal speed of the car in the inertial coordinate system.

The side slip angles of the front and rear tires α_t and are defined in the Figure 2. $\beta_t$and β_s are the angles formed by the speed vector and the longitudinal speed direction. The following relationships may be used to calculate β_t, β_s using small angle approximations as follows:

$$\beta_t = \frac{v_y + a.\dot{\psi}}{v_x} \qquad (5)$$

$$\beta_s = \frac{v_y - b.\dot{\psi}}{v_x} \qquad (6)$$

The test results show that the lateral force of the tire increases or decreases with the side slip angle when the angle is still small. Therefore, the lateral force of the tire on the front wheel of the car may be expressed as follows:

$$F_{y_t} = 2.C_{\alpha t}.(\delta_t - \beta_t) \qquad (7)$$

Similarly, the lateral tire force for the rear wheels is expressed as:

$$F_{y_s} = 2.C_{\alpha s}.(-\beta_s) \qquad (8)$$

In this context, $C_{\alpha\ t}$ and $C_{\alpha\ s}$ represents the lateral stiffness of the front and rear wheels. The steering angle of the front wheels is denoted by δ_t , while the steering angle of the rear wheels is represented by δ_s , which is assumed to be zero (i.e., δ_s = 0). The variables α_t and α_s are the side slip angles of the front and rear wheels respectively.

By incorporating equations (5), (6), (7), and (8) into equations (2) and (3), the state-space model that characterizes the yaw motion of the car can be articulated as a function of the car's yaw rate and sideslip, as detailed below:

$$\frac{d}{d_t}\begin{bmatrix}\dot{y}\\ \psi\\ \dot{\psi}\\ Y\end{bmatrix} = \begin{bmatrix} -\frac{2.C_{\alpha t}+2.C_{\alpha s}}{m.\dot{x}} & 0 \\ 0 & 0 \\ -\frac{2.a.C_{\alpha t}-2.b.C_{\alpha s}}{I_z.v_x} & 0 \\ 1 & v_x \\ 0 & -v_x - \frac{2.a.C_{\alpha t}-2.b.C_{\alpha s}}{m.v_x} \\ 1 & 0 \\ -\frac{2.a^2.C_{\alpha t}+2.b^2.C_{\alpha s}}{I_z.v_x} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix}\dot{y}\\ \psi\\ \dot{\psi}\\ Y\end{bmatrix} + \begin{bmatrix}\frac{2C_{\alpha t}}{m}\\ 0\\ \frac{2aC_{\alpha t}}{I_z}\\ 0\end{bmatrix}\delta_t(t) \qquad (9)$$

In equation (9), a nonzero reference can be set for the car's lateral trajectory. This equation uses the difference between the specified lateral reference trajectory, Y^{ref} , and the car's actual lateral trajectory as its input:

$$e(t) = Y^{ref}(t) - Y(t) \qquad (10)$$

METHODS

This paper explores the challenge of decision making and trajectory planning to ensure safe driving, focusing on controlling the motion of the master car in various driving situations, such as two-lane roads in the same direction, two-lane roads in opposite directions, and other complex environments.

Figure 3depicts a traffic situation involving the host car and a leading car. The leading car is located in the same lane as the host car, while the adjacent lane remains vacant. The host car is anticipated to adjust its actions (such as maintaining its lane, overtaking, etc.) in response to potential safety hazards within its driving environment. The main goal of this proposed paper is to enable the host car to:

Objective (i): Track the midpoint of the lane.

Objective (ii): Drive at the desired speed.

Objective (iii): Ensure environmental constraints and drivetrain limits.

Objective (iv): Ensure safety with the car ahead.

The host car consists primarily of two layers, Layer 1: the decision-making layer and Layer 2: the lateral trajectory planning layer. Each layer considers the status of the road (occupied or free) and the inter-spacing between the host car and its front car to prevent entry into the danger zone (red zone) depicted in Figure 3. In this paper, the host car considers a scenario where the spacing between the host car and the car in front is d_2 , indicating an approach to the red zone, while the adjacent lane is unoccupied. In Layer 1, the host car will perform a lane change then a lane staying in the adjacent lane by layer 2. Here, the red zone represents the safe distance limit, and the navigable zone of the host car is determined by a reference trajectory, which will be discussed in this section.

Safety distance model

For safety purposes, the host car must navigate within a collision-free corridor by controlling the safety distance with its front car, the safety spacing delimited by the red zone. The safety spacing is defined as the minimum following spacing that the following car must maintain to avoid colliding with the front car as the speed of the car in front is less than the speed of the car in back. A schematic illustrating the safety spacing is depicted in Figure 3.

The safety distance can be approximated as [25]:

$$d_{safe} = d_0 + \frac{v_h^2}{2a_h} + (v_h - v_p)t_h - \frac{v_p^2}{2a_p} \qquad (11)$$

where, the following car (host car) is moving at velocity v_h, the previous car (front car) is moving at velocity v_p .a_p, a_h are the accelerations of the previous car

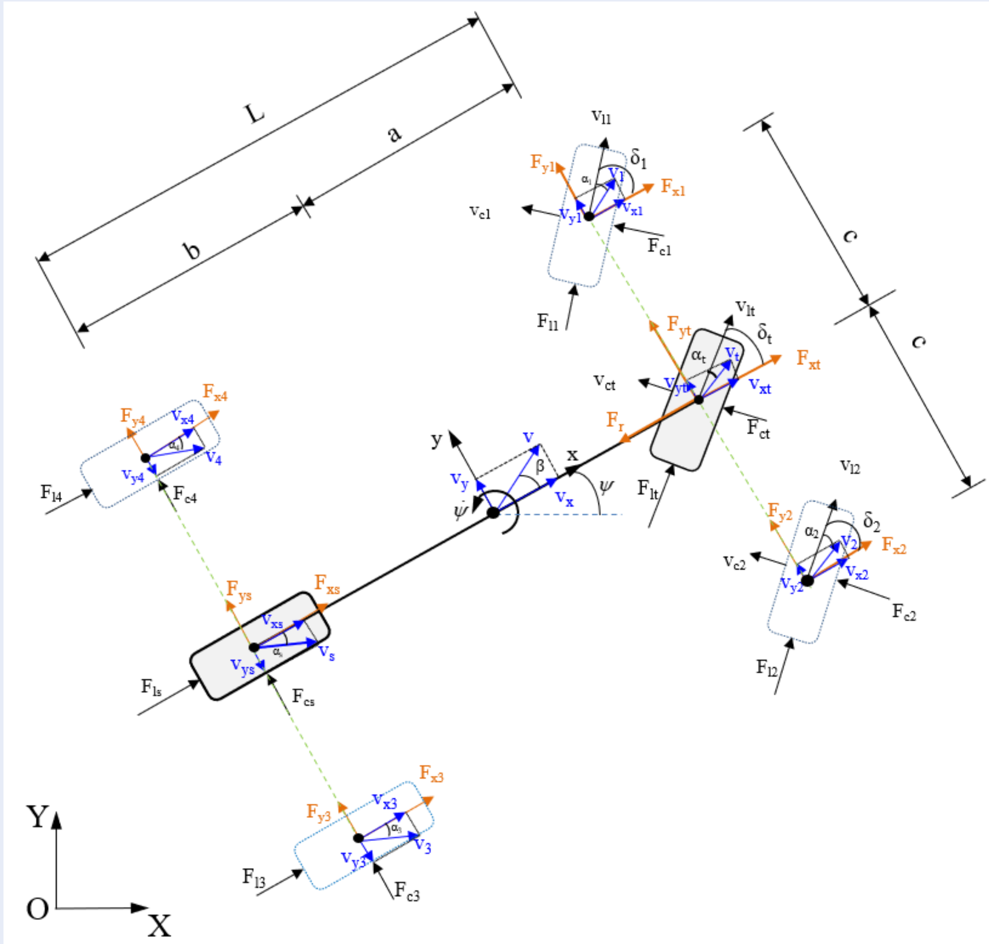

Figure 1: The car dynamicsmodel in multiple coordinate systems [24]

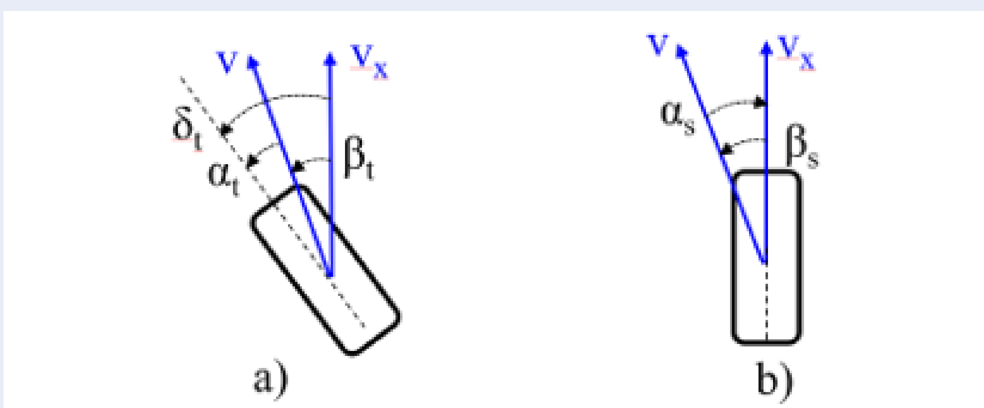

Figure 2: Tire slip-angle: a) side slip angle of front tire, b) sideslip angle of rear tire

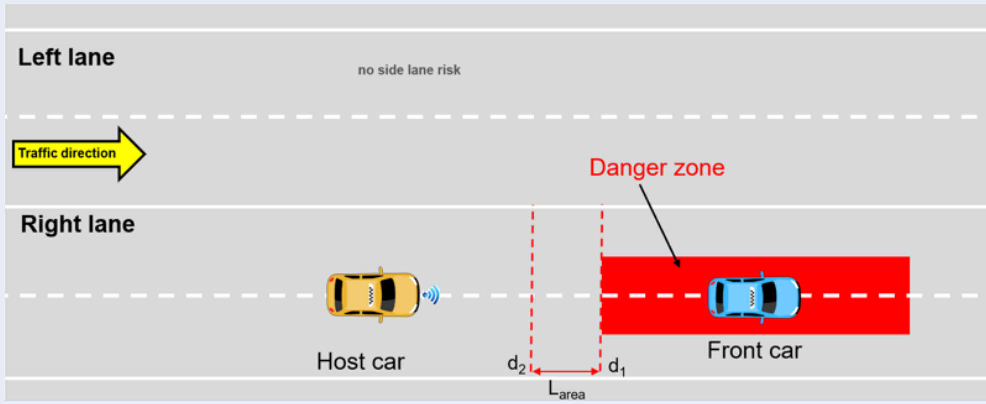

Figure 3: The danger zones of front car for the host car with L_{area} isthe length of the lane changing area

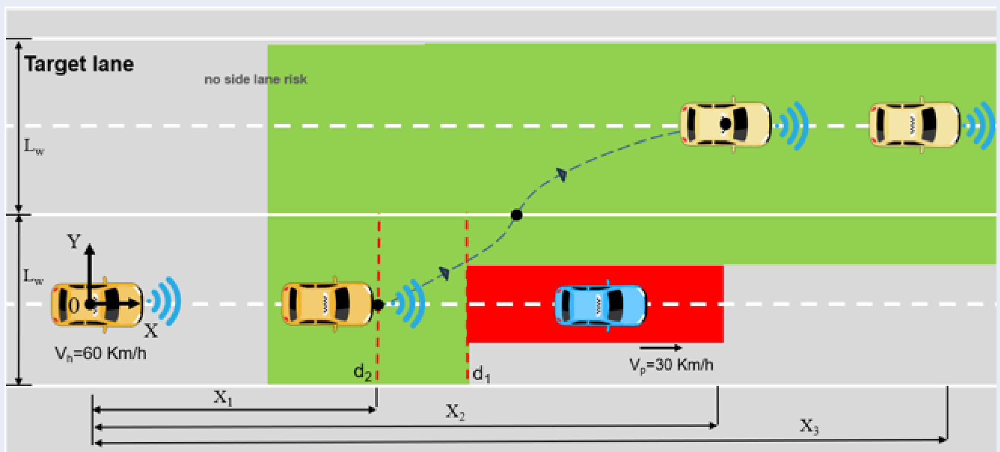

Figure 4: The host car's navigation zone delimited by the green zone and generatedreference trajectory zones.

and host car respectively. t_h is the safety time. d_0 indicates the relative spacing between the previous car and the host car after coming to a halt. Therefore, the minimum following distance d_1 should be specified as follows:

$$d_1 = 2 + \frac{v_h^2}{2a_h} + (v_h - v_p)t_h - \frac{v_p^2}{2a_p} \qquad (12)$$

The maximum following distance is influenced by driver behavior and generally changes with the car's speed. In this context, we define the maximum following distance d_2 as follows:

$$d_2 = 10 + \frac{v_h^2}{2a_h} + (v_h - v_p)t_h - \frac{v_p^2}{2a_p} \qquad (13)$$

Trajectory planning problem description

The host car's navigable zone is indicated by a green area as in Figure 4, with the shape of the reference trajectory continuously optimized in real-time to adapt to driving conditions, environmental constraints and the subject.

In the present work, it is assumed that:
Assumption (i): The host car is equipped with sensors that measure its spacing and position.
Assumption (ii): Constant longitudinal velocity.
Assumption (iii): The road is straight.
Assumption (iv): All tires have the same characteristics.
Assumption (viii): External disturbances are bounded.

To ensure a trajectory with zero jerk at the extremes of the maneuver, which would enhance comfort, a 5^{th} order polynomial lane change trajectory is proposed. This trajectory possesses the capability to generate a smooth and suitable path for the car during the lane change process. Choosing the appropriate trajectory function can improve the autonomous car's capability to execute a safe and efficient lane change. Therefore, the imposed reference is a trajectory to be followed by the host car as follows:

$$\begin{aligned}
&Y(X) = 0, \quad X \in [0, X_1] \\
&Y(X) = a_0 + a_1(X - X_1) + a_2(X - X_1)^2 + \\
&a_3(X - X_1)^3 + a_4(X - X_1)^4 + a_5(X - X_1)^5 \\
&X \in [X_1, X_2] \\
&Y(X) = L_W, \quad X \in (X_2, X_3]
\end{aligned} \quad (14)$$

$$\begin{cases}
Y(X) = 0, & at\ X = X_1 & (15a) \\
\dot{Y}(X) = 0, & at\ X = X_1 & (15b) \\
\ddot{Y}(X) = 0, & at\ X = X_1 & (15c) \\
Y(X) = L_w, & at\ X = X_2 & (15d) \\
\dot{Y}(X) = 0, & at\ X = X_2 & (15e) \\
\ddot{Y}(X) = 0, & at\ X = X_2 & (15f)
\end{cases} \quad (15)$$

The unknown coefficients are obtained by solving Equation (15), which represents the solution to the system of equations:

$$a_0 = 0, a_1 = 0, a_2 = 0, a_3 = \frac{10L_W}{L_X^3}, \quad a_4 = \frac{-15L_W}{L_X^4}, a_5 = \frac{6L_W}{L_X^5} \quad (16)$$

The obstacle-free path enables the car to transition from its current position to a designated target position while avoiding collisions. This path is defined as follows:

$$Y^{ref} = \begin{cases}
0, \ \forall X \in [0, X_1) \\
\frac{10L_W}{L_{X^3}}(X - X_1)^3 - \frac{15L_W}{L_{X^4}}(X - X_1)^4 + \\
\frac{6L_W}{L_{X^5}}(X - X_1)^5, \ \forall X \in [X_1, X_2] \\
L_W, \ \forall X \in (X_2, X_3]
\end{cases}$$

(17)

Y^{ref} can also be expressed as a function of time by defining t_{scc} as follows: $t_{scc} = \frac{L_X}{V_H}$ with $L_X = X_2 - X_1$ in (17), such that:

$$Y^{ref} = \begin{cases}
0, \ \forall 0 \leq t < t_1 \\
\frac{10L_W}{t_{scc^3}}(t - t_1)^3 - \frac{15L_W}{t_{scc^4}}(t - t_1)^4 + \\
\frac{6L_W}{t_{scc^5}}(t - t_1)^5, \ \forall t_1 \leq t \leq t_2 \\
L_W, \ \forall t_2 < t < t_2
\end{cases} \quad (18)$$

The host car is set to change lanes along the centerline of the left lane, ensuring that its final lateral position L_W complies with the lane width. We assume that the longitudinal velocity of the host car remains constant throughout the maneuver, denoted as $V_H = v_x(X)$. The lateral velocity and lateral acceleration, represented by v_y and a_Y respectively, are both assumed to be zero. We aim to calculate the duration of the lane change, t_{scc}, while $t_1, \ldots, t_3$ denote the times corresponding to positions $X_1, \ldots, X_3$ in the XOY coordinate system and the longitudinal distance required to form the trajectory. However, the values of these times can be arbitrary in some studies.

Safety Constraints

Position limitation

The host car ensures that it moves within the lane, we set the world standard lane width as 3.5m [26], less than the longitudinal displacement ; longitudinal speed as 60km/h. We set the maximum longitudinal distance as 64m, because changing lanes at short distances affects safety.

$$\begin{aligned}
&0 \leq Y \leq L_W, \ \forall X_1 \leq X \leq X_2 \\
&0 \leq X \leq L_X, \ \forall X_1 \leq X \leq X_2
\end{aligned} \quad (19)$$

where Y and X represent the horizontal and vertical positions of the host car's lane (direction of car movement), respectively.

Speed limitation

It is crucial that the speed of the front car remains lower than that of the host car on the current road, ensuring safe and feasible maneuvering. Additionally, the front car's longitudinal speed must always remain positive to maintain continuous forward motion [27].

$$0 \leq V_P \leq V_H \quad (20)$$

where the longitudinal speed V_P as 30 km/h.

Comfort

The acceleration values in both longitudinal and lateral directions should be kept minimal to provide a smooth and comfortable driving experience [24]:

$$\begin{aligned}
&-a_{x,max} \leq X(t) \leq a_{x,max} \\
&-a_{y,max} \leq Y(t) \leq a_{y,max}
\end{aligned} \quad (21)$$

Where $a_{x,max}$ is the maximum allowable longitudinal acceleration as 2 m/s^2, and $a_{y,max}$ is the maximum allowable lateral acceleration as 2 m/s^2.

Actuator limitation

Since the longitudinal speed is significantly greater than the lateral velocity, the steering angle is constrained by the actuator of the steering system [24]:

$$\widetilde{\delta_t}(t) = sat(\delta(t)) \begin{cases}
\delta(t) \ for \ |\delta_t| \leq 0.1845 \\
0.1845 \ for \ \delta_t > 0.1845 \\
-0.1845 \ for \ \delta_t < 0.1845
\end{cases} \quad (22)$$

CONTROLLER DESIGN

MPC is an advanced control strategy that effectively manages constraints by embedding them within the design process. A key advantage of MPC lies in its ability to incorporate a system model that accounts for both dynamic and static interactions among inputs, outputs, and external disturbances while ensuring constraints on inputs and outputs are properly handled. Consequently, the control framework leverages MPC algorithms to regulate the lateral dynamics of the car.

This section provides a conciseooverview of the MPC design methodology, where the system's behavior is predicted using a state-space model, enabling precise and efficient trajectory planning:

$\chi_i(k+1) = A_i\chi_i(k) + B_i u_i(k) \quad (23)$

$Z_i(k) = C_i.\chi_i(k) \quad (24)$

here, χ_i=[y,ψ,ψ,Y]^T represents the state vector of subsystem *i*, including velocity, yaw angle, yaw rate, and lateral position. Z_i denotes the forecasted future outputs of subsystem *i*, u_i indicates the control command applied to the system, A_i is the state matrix, defining the evolution of the subsystem's state, while B_i and C_i are matrices representing the input and output relationships within subsystem *i*.

The predictive control problem can be calculated as follows: starting with an initial state $\chi_i(0)$= χ_i(k) χ_i, the goal is to determine a finite sequence of inputs$\{u_0,\ u_1,\ u_2, \ldots,\ u_{N-1}\}$ that minimizes the finite horizon cost function:

$$J_{MPC}\left(\chi_i\left(k\right), u_i\left(k\right)\right) = \chi_N^T P \chi_N + \sum_{i=0}^{N-1}\left(\chi_N^T Q \chi_N + u_i^T R u_i\right) \quad (25)$$

In this context, *N* represents the prediction horizon, *P* and *Q* are the weight matrices associated with the system's states, and *R* is the weight matrix of the control command. The system's performance is influenced by selecting appropriate values for $R \geq 0$, $Q \geq 0$, *P* and *N*. The vector χ_i denotes the predicted state sequence χ_i (k+1) for all *i=0, 1, …, N*. The prediction equation for the system state is derived using equations (23) and (24), resulting in:

$$\Upsilon = \Phi\chi_i\left(k\right) + \Gamma U_i\left(k\right) \quad (26)$$

Thus, the equation (25) may be rewritten in matrix form:

$$J_{MPC}\left(\chi_i\left(k\right), U_i\left(k\right)\right) = \chi_N^T Q \chi_N + \Upsilon^T \Xi \Upsilon + U_i^T \Lambda U_i \quad (27)$$

in which $\Upsilon = [\chi_1, \chi_2, \ldots, \chi_N]^T, U_i = [u_0, u_1, \ldots, u_{N-1}]^T, \Lambda = dig\{R, \ldots, R\}, \Xi = dig\{Q, \ldots, Q, P\}$

$$\Gamma = \left(\begin{array}{cccc} B_i & 0 & \ldots & 0 \\ A_i B_i & B_i & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ A^{N-1}{}_i B_i & A^{N-2}{}_i B_i & \ldots & B_i \end{array}\right]$$

ILLUSTRATIVE RESULTS AND DISCUSSION

This paper primarily focuses on path planning and tracking control, where the intelligent car utilizes onboard sensors to collect data on obstacles and road conditions. As the host car nears an obstacle, its trajectory planner generates a new reference path to avoid a collision, as defined in Eq. (17). To evaluate the performance of the proposed safe driving algorithm, a numerical simulation is implemented in Simulink environment. The scenario involves a two-lane, one-way road with host car, previous car traveling in the same direction along the lane centerlines. The results, discussed in Section 3, show both cars moving at constant speeds: the host car at v_H = 60km/h and the front car at v_p = 30 km/h. Additionally, it is assumed that no other cars are present in the left lane. In this setup, the host car maintains a safe following distance and remains centered in the lane.

The sensors detect a front car in the same lane as the host car. As the host car gets closer to this front car, the distance between them is referred to as d_2. When this distance is reached, the host car decides to execute a lane change and begins planning its trajectory to prevent a collision. The trajectory planner generates a new reference path to navigate around the obstacle. The predictive controller for the system was implemented with the following parameters: prediction horizon *N = 10*, Q = $8I_5$, P = $10I_5$, and R = 0.02, where I_5 indicates the 5^{th} order identity matrix. These controller parameters *Q, R, P*, and *N* were given to ensure a response that exhibits strong performance. The car parameters used in the simulations are outlined in Table 1.

The control structure for the lateral dynamics, as illustrated in Figure 5, consists of two key outputs. The first output represents the lateral position obtained from the car's nonlinear model, while the second output corresponds to the lateral position derived from the simplified lateral dynamics model. The car's lateral behavior is formulated based on equations (1) through (9), ensuring consistency between theoretical modeling and real-world implementation. In this configuration, the steering angle of the front tires serves as the primary control input, directly influencing the car's lateral motion and overall stability.

The results of the host car's lateral control are illustrated in Figure 8, which represents the model of the lateral dynamics. From Figure 8, it can be observed that the host car successfully follows the reference trajectory while safely avoiding a collision with the front car. Additionally, the deviation between the

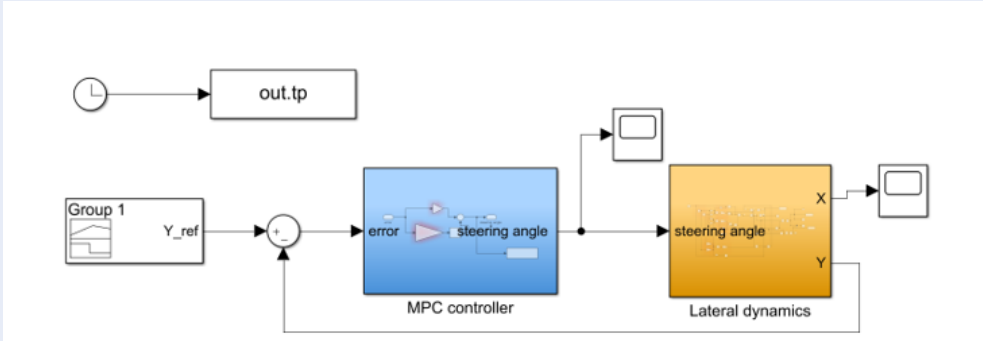

Figure 5: Lateral control structure.

Table 1: Car parameters [28]

Symbol	Value	Unit
m	1450	Kg
C_{at}	80 000	N/rad
C_{as}	100 000	N/rad
a	1.3	m
b	1.45	m
I_z	1920	kg. m2

planned path and the actual path is depicted in Figure 9, providing insight into the accuracy of the trajectory tracking performance. Overall, the signals resulted after simulation can be observed that location error of the host car converges to zero value. The peak value of the location error generated by the MPC algorithm is about 0.00022 m. It can be observed that the MPC algorithm can reduce the peak value of the location error of the host car. This implies that the MPC controller implemented with the lateral model has higher performance for the nonlinear model.

Figure 6 illustrates the steering angle of the front tire (control command) for the host car. This illustration shows that the control signal adheres to the constraints specified in equation (22), with δ_{max} = 0.174 and δ_{min} = -0.174. The car yaw angle is depicted in Figure 7, it changes when the car makes a lane change, meaning it changes when the steering angle is different from 0.

Figure 8 depicts the reference trajectory generated by the trajectory planner alongside the actual trajectory of the host car. The results confirm that the trajectory planner effectively creates an obstacle-free path, while the car's lateral controller ensures precise tracking of the reference trajectory. Furthermore, the lateral position constraints outlined in equations (19), (20), and (21) are successfully maintained, demonstrating the effectiveness and reliability of the proposed control strategy. Additionally, sub-figures (a, b, c) in Figure 10 illustrate that the car's trajectory planners are capable of generating feasible trajectories for high-speed cars, allowing them to maneuver around fixed obstacles.

CONCLUDING REMARKS

This paper presents the development of a model-based lateral MPC controller focused on trajectory control for car lateral positioning. The main aim is to achieve accurate lateral control for a model that closely mimics real car dynamics. The control algorithm effectively maintains the car's lateral position, ensuring it follows the reference trajectory while meeting control signal constraints. Simulations conducted in Matlab/Simulink demonstrate that the trajectory planner successfully generates a collision-free path, even when a leading car is present.

Future research will focus on designing two model-based MPC controllers to handle both longitudinal and lateral dynamics, aiming to create a versatile control system adaptable to various driving scenarios, such as two-lane roads in the same direction, two-lane roads in opposite directions, and similar scenarios.

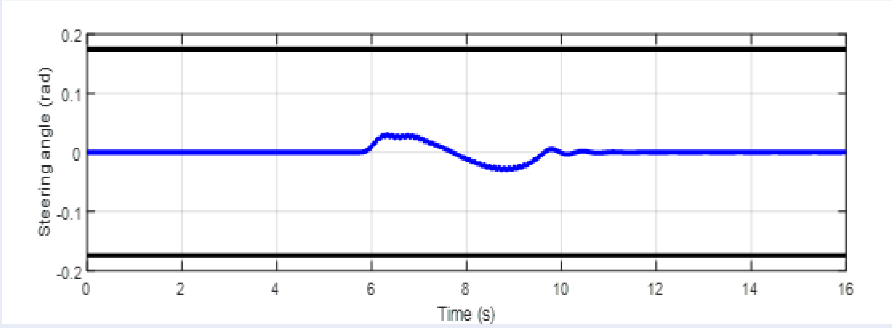

Figure 6: The steering angle ofhost car.

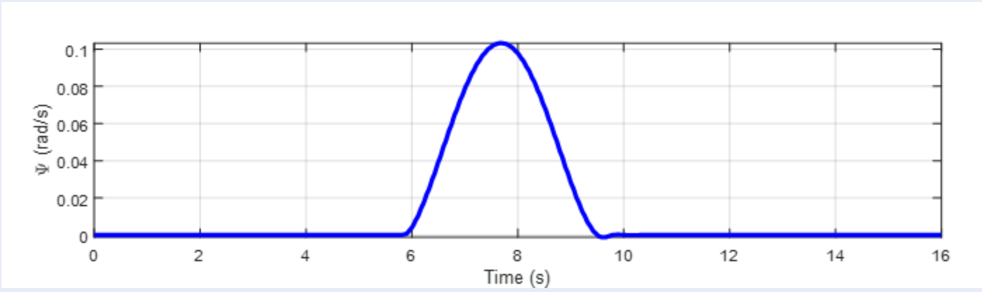

Figure 7: The yaw angle of host car.

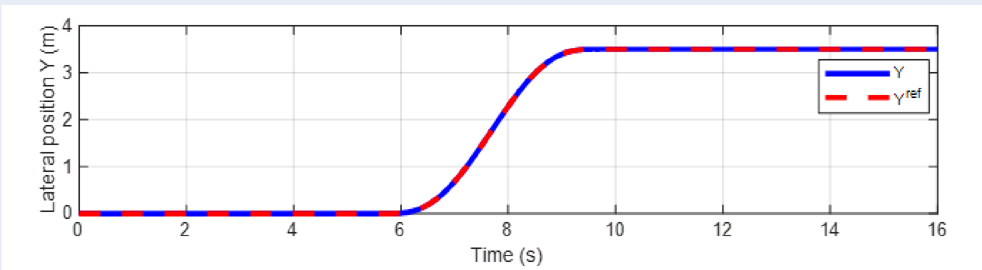

Figure 8: Planned path and actual path.

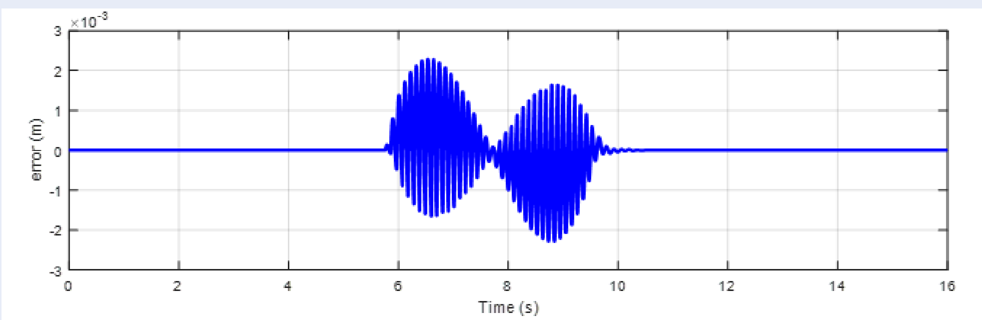

Figure 9: The deviation between planned path and actual path.

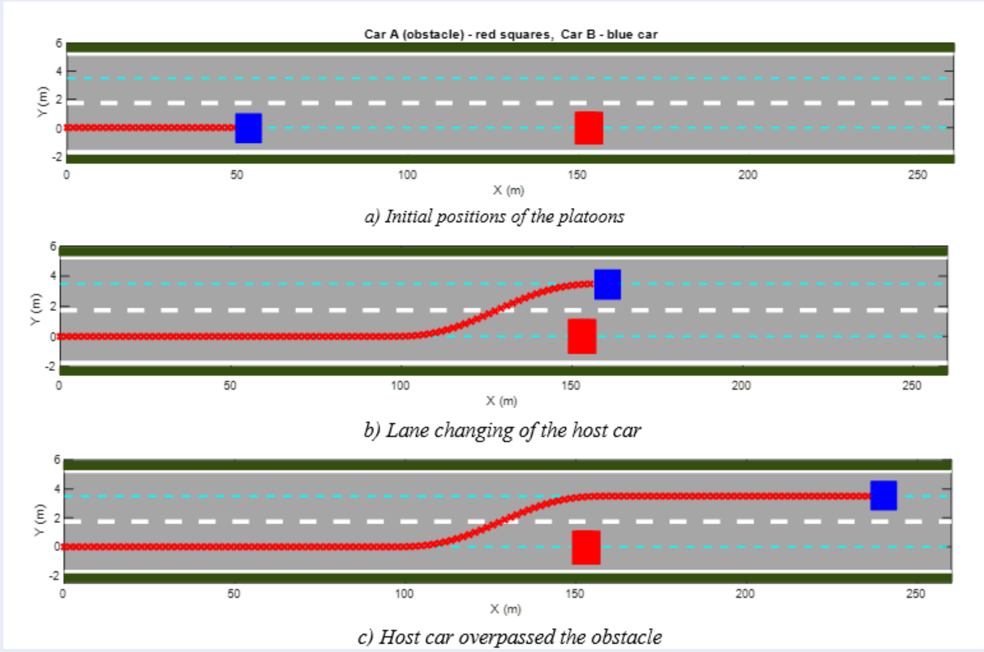

Figure 10: Snapshots of the trajectory of the host car and obstacle overpassing.

COMPETING INTERESTS

There is no conflict of interest.

AUTHORS' CONTRIBUTIONS

Duc Lich Luu: conceptualization, methodology, formal analysis, validation, writing—review and editing. **Thanh-Long Phan, Tien Thua Nguyen**: reviewing and editing. **Huynh Vinh Quang, Nguyen Dac Thanh Dat**: writing – original draft preparation. All authors have read and agreed to the published version of the manuscript.

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Bài nghiên cứu

Thiết lập quỹ đạo chuyển làn cho lái xe tự động dựa vào đa thức bậc năm

Lưu Đức Lịch*, **Phan Thành Long, Nguyễn Tiến Thừa, Huỳnh Vinh Quang, Nguyễn Đắc Thành Đạt**

TÓM TẮT
Bài báo này tập trung vào việc kiểm soát chuyển động của xe tự hành, đặc biệt là khi thực hiện các thao tác chuyển làn an toàn. Giải pháp được đề xuất tích hợp Kiểm soát dự đoán mô hình (MPC) với trình lập kế hoạch quỹ đạo đa thức bậc năm để xử lý việc chuyển làn và tránh va chạm trong môi trường lái xe năng động. Ưu điểm chính của phương pháp này là yêu cầu tài nguyên tính toán tối thiểu, giúp phương pháp này phù hợp để triển khai theo thời gian thực trong khi vẫn duy trì hiệu suất cao trong điều kiện giao thông phức tạp. Bài báo bắt đầu bằng cách phát triển một mô hình động phi tuyến tính của xe, nhấn mạnh vào động lực ngang, yếu tố rất quan trọng để lập kế hoạch và kiểm soát chuyển động của xe trong quá trình chuyển làn. Mô hình tính đến các thông số quan trọng như tốc độ lệch, lực ngang và góc lái. Trình lập kế hoạch quỹ đạo được thiết kế để tính toán đường đi tối ưu, không va chạm để xe đi theo khi chuyển làn hoặc vượt xe khác, đảm bảo xe luôn nằm trong giới hạn an toàn, chẳng hạn như duy trì khoảng cách thích hợp với các xe đi trước. Một khía cạnh mới của giải pháp được đề xuất là tích hợp việc ra quyết định với lập kế hoạch quỹ đạo. Hệ thống tính toán khoảng cách an toàn với xe đi trước bằng cách sử dụng số liệu thời gian va chạm và thời gian giữa các xe. Các số liệu này cho phép xe quyết định giữ nguyên làn đường hay bắt đầu chuyển làn, dựa trên tính an toàn của thao tác. Sau khi đưa ra quyết định, trình lập kế hoạch quỹ đạo sẽ tạo ra một đường tham chiếu mới, đảm bảo chuyển làn an toàn và êm ái, ngay cả khi có chướng ngại vật. Hiệu quả của hệ thống được đề xuất được chứng minh thông qua các mô phỏng mở rộng trong nhiều tình huống lái xe khác nhau. Các mô phỏng này cho thấy xe có thể thực hiện thành công việc chuyển làn và vượt xe mà không va chạm với các xe khác, đồng thời vẫn duy trì sự thoải mái và giảm thiểu lỗi điều khiển. Kết quả mô phỏng xác nhận rằng hệ thống điều khiển dựa trên MPC, kết hợp với trình lập kế hoạch quỹ đạo đa thức, cung cấp giải pháp đáng tin cậy và hiệu quả cho việc lập kế hoạch và kiểm soát quỹ đạo theo thời gian thực trong lái xe tự động.
Từ khoá: Điều khiển dự đoán theo mô hình, lập kế hoạch quỹ đạo, tránh va chạm, ra quyết định, động lực học ô tô, điều khiển ô tô.

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