

The application of multivariable membership functions to the fuzzy neural model

Bui Truong An^{1,*}, Pham Tuan Anh¹, Pham Thi Nguyen²



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ABSTRACT

In recent years, fuzzy neural systems have become increasingly popular due to their powerful learning and interpreting capabilities. In the field of control, the fuzzy neural system is superior to other intelligent systems. In addition, the theory of the combination of neural networks and fuzzy logic is also used in many other fields such as prediction, simulation, decision support, etc. However, most neural systems are Fuzzy systems used today are a combination of neural networks and univariate membership functions in fuzzy theory. These functions have the advantage of being simple and easy to set up, but with that is a lack of interpretability for complex objects. For objects that need to be described by two or more quantities, unidirectional membership functions are not able to represent it. Application of multivariable membership function is necessary in this case. The application of multivariable membership functions encounters many barriers due to their complexity, the algorithms for applying multivariable membership functions are sketchy and have not fully promoted its advantages. In this article, we will introduce a method for applying multivariable Gaussian membership function that allows to improve simulation performance compared to previously introduced methods.

Key words: fuzzy set, multivariable membership functions, Gaussian functions, fuzzy neural model

1 INTRODUCTION

2 Neutral networks consist of a large number of simple processing elements (neurons) that are interconnected, so when processing information in parallel, there is a huge computing power. However, the knowledge accumulated by the neural network is distributed among all its elements, which makes them practically inaccessible to the observer.

3 Fuzzy logic control systems do not have this limitation. However, control knowledge is required at the design stage of the control module and must come from experts, and therefore the fuzzy logic control system is not capable of learning.

4 Combining both approaches allows you to create a system that has both the ability to train a neural network and enhance the intellectual abilities of the system with fuzzy decision rules inherent in the "human" way of thinking.

5 Such neuro-fuzzy systems are very diverse and are increasingly being improved in accordance with the development of neural network learning algorithms. Among them are gradient descent methods¹. The disadvantage of these algorithms is that they are slow if the definition of the training step is not satisfactory, and converge easily to local minima. Population algorithms solve these problems and are effective

6 in optimizing a large space, divided into two groups, including evolutionary algorithms and swarm algorithms. The genetic algorithm (GA) belongs to the group of evolutionary algorithms based on such genetic processes as selection, mutation, and exchange². Another algorithm belonging to the group of evolutionary algorithms is the differential evolution algorithm³ also inspired by biology such as GA, the difference is that a mutant element is created by adding an efficiency number between two elements with previous generations. The swarm algorithm group often draws ideas from animal behavior, such as particle swarm algorithm⁴, ant colony algorithm⁵, bee swarm algorithm⁶, cuckoo search algorithm⁷. There are also support vector algorithms⁸ and extreme machine learning algorithms⁹.

7 Most of the above works are built on the basis of one-dimensional membership functions, such as Gaussian, Bell, Triangular. The limitation of this approach is the complexity of the model in terms of the number of rules, which increases exponentially with the number of inputs (spatial curse).

8 As an effective solution to the above problem, the use of multivariable membership functions in a fuzzy inference system is proposed. In Abonyi *et al.* (2001)¹⁰, a fuzzy model with triangular multivariable membership functions is introduced; these membership func-

¹Institute of Information Technology, AMST, Ho Chi Minh City, Vietnam.

²Ho Chi Minh City University of Natural Resources and Environment, Ho Chi Minh City, Vietnam

Correspondence

Bui Truong An, Institute of Information Technology, AMST, Ho Chi Minh City, Vietnam.

Email: buitruonganmta92@gmail.com

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tions are obtained by Delaunay triangulation of their characteristic points. The problem with this method is that each fuzzy set is created by linking "nodes" in space. For a multidimensional space, the number of "nodes" required to represent a fuzzy set increases rapidly, resulting in a large number of variables to describe the membership function. Compared to triangular multivariable Delaunay functions, multivariable Gaussian functions require fewer variables to describe a fuzzy set¹¹⁻¹³. In Kang *et al.* (2007)¹¹, membership functions are identified using the clustering algorithm. Distance calculation is performed on input and output variables, so data in the same group may not have the same output properties. Data with similar outputs may be located in different clusters due to the greater distance in the input space. This lack of association reduces the explanatory power of the generated fuzzy system.

In Lemos *et al.* (2010)¹², Pratama *et al.* (2013)¹³, multivariable Gaussian membership functions are used in developing fuzzy models. The algorithms in the above works are used to develop a fuzzy inference system based on a sequential set of input data. This leads to the fact that in the presence of training data sets, their high accuracy is not guaranteed.

In Basil *et al.* (2019)¹⁴, multivariable Gaussian membership functions are used with incomplete covariance matrices, which is essentially another expression for using one-dimensional Gaussian membership functions. The use of such membership functions provides neither the advantage of the number of fuzzy rules nor the decomposition error reduction.

Most of the algorithms for determining the parameters of fuzzy membership functions in the above works are developed on the basis of algorithms for the synthesis of fuzzy rules for one-dimensional membership functions.

The fuzzy rules generated by these algorithms often overlap and cannot act as independent rules. The overlap of fuzzy rules in fuzzy systems does not allow assessing the reliability of individual fuzzy rules and at the same time creates limitations in extracting knowledge from fuzzy systems. When applying a fuzzy neural system based on a multivariable membership function to decision support systems, the ability to operate independently of fuzzy rules is very important, since it allows you to evaluate the accuracy of a solution given on the basis of individual fuzzy rules. Therefore, the task of constructing a multivariable membership function with fuzzy rules capable of independent operation is relevant.

MULTIVARIATE MEMBERSHIP FUNCTION

To represent multidimensional fuzzy sets, we use multivariable membership functions. Multivariable membership functions are also divided into linear and non-linear. A commonly used linear multivariate membership function is a triangular multivariate membership function.

A linear multivariable membership function is obtained by Delaunay triangulation¹⁵ of their characteristic points.

Like one-dimensional linear membership functions, multivariable linear membership functions have limited flexibility in setting parameters, which complicates the formation of complex dependencies.

Compared to linear multivariate membership functions, non-linear multivariate membership functions, especially multivariate Gaussian functions, are more widely used.

In general, the one-dimensional Gaussian membership function uses an exponential function to project the distance D from a point x in space to the center of the fuzzy set d_1 on the interval $[0,1]$ such that the distance between x and the greater d_1 , the smaller the value of the membership function at the point x and vice versa. The multivariate Gaussian membership function also uses the same principle:

$$X(x) = e^{-D^2(x)} \tag{1}$$

where: D is the distance from x to the center C of the fuzzy set, $x = (x_1, x_2, \dots, x_n)$ are the variables of the multivariable membership function, where n is the number of space dimensions.

The distance D can simply be defined as the Euclidean distance in the space x and C :

$$D = \sqrt{\sum_{i=1}^n (x_i - C_i)^2} \tag{2}$$

where x_i is the i -th variable of the multivariable membership function corresponding to the i -th dimension in space;

C_i - i -th coordinate of the fuzzy set center.

The limitation of using Euclidean distance is that the extent is the same in all directions (Figure 1). This reduces the spatial separation of fuzzy sets.

A generalization of Euclidean distance, called normalized Euclidean, allows you to narrow or expand the membership function in a direction parallel to the coordinate axes (Figure 2):

$$D = \sqrt{\sum_{i=1}^n \left(\frac{x_i - C_i}{\sigma_i} \right)^2} \tag{3}$$

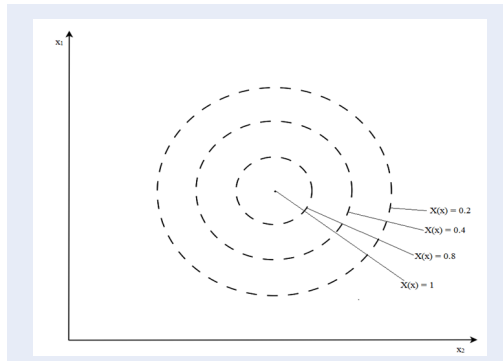


Figure 1: Spatial distribution of multivariate Gaussian membership function based on Euclidean distance

149 where $\sigma_1, \sigma_2, \dots, \sigma_n$ - are the expansion coefficients in
 150 dimensions parallel to the coordinate axes.
 151 For expansion or contraction in an arbitrary direc-
 152 tion, it is proposed to use the distance based on the
 153 idea of Mahalanobis¹⁶, according to which:

$$D(x) = \sqrt{(x - C)^T S^{-1} (x - C)} \quad (4)$$

154 where: $x = [x_1, x_2, \dots, x_n]$ - the matrix of variables of
 155 the membership function has the size $(1 \times n)$, n - the
 156 number of spatial dimensions; $C = [C_1, C_2, \dots, C_n]$ -
 157 matrix of coordinates of the center of the fuzzy set X
 158 of size $(1 \times n)$.
 159 S is a matrix of expansion coefficients (variation) of
 160 size $(n \times n)$; S^{-1} is the inverse of S .

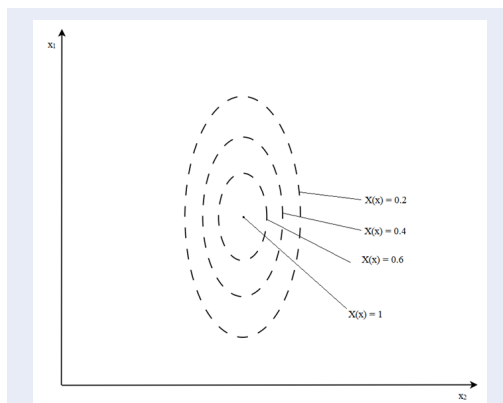


Figure 2: Spatial distribution of Gaussian membership function with two variables based on normalized Euclidean distance

161 On Figure 3 shows the spatial distribution of the
 162 Gaussian membership function based on the Maha-
 163 lanobis distance with the parameters:

$$C = [5 \ 5], \quad S = \begin{bmatrix} 10 & 6 \\ 6 & 5 \end{bmatrix} \quad \text{with cut-offs} \quad 164$$

$$\alpha = \{0.8 \ 0.6 \ 0.4\} \quad 165$$

The matrix S is chosen and transformed to be a posi-
 166 tive definite matrix such that the value under the rad-
 167 ical is always positive for $x - C$ values. 168

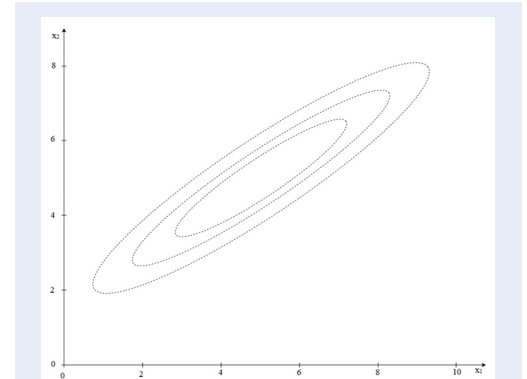


Figure 3: Spatial distribution of a two-variable Gaussian membership function based on the Mahalanobis distance

The Mahalanobis distance is a generalized form of the
 169 normalized Euclidean distance. If S is the identity ma-
 170 trix, then the Mahalanobis distance becomes equal to
 171 the Euclidean distance. If S is a diagonal matrix and
 172 has the value 173

$$S = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \sigma_n^2 \end{bmatrix},$$

then the Mahalanobis distance becomes the normal-
 174 ized Euclidean distance. 175

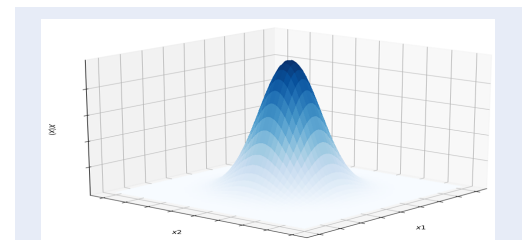


Figure 4: Membership function with two variables

Fuzzy sets with multivariable Gaussian membership
 176 functions (Figure 4) have more flexible spatial div-
 177 ision than one-dimensional membership functions,
 178 and at the same time have a small number of param-
 179 eters and are easier to implement than a triangular 180

181 multivariable membership function. Therefore, in re-
 182 cent years it has often been used for the synthesis of
 183 fuzzy inference systems in various fields¹⁷.

184 **METHOD**

185 **Methods to combine two membership func-**
 186 **tions**

187 Shrinath. G. A., Plamen. P. A., Lughofer E., Bouchot
 188 J. L. and Shaker A. proposed a method for combining
 189 two fuzzy sets as follows¹³:

$$C_+ = (\max(U) + \min(U)) / 2 \quad (5)$$

$$S_+ = (\max(U) - \min(U)) / 2 \quad (6)$$

190 where C_+ , S_+ are the coordinates of the center shift
 191 point and the matrix of coefficients of the latitudes
 192 of the total fuzzy set, $U = \{C_1 \pm \sigma_1, C_2 \pm \sigma_2\}$, where
 193 C_1, C_2 are the coordinates of the points of the cen-
 194 ter shift of the component membership functions, $\sigma_1,$
 195 σ_2 are the width parameters of the fuzzy sets on the
 196 α -section. The coverage of the total fuzzy set by this
 197 method covers the coverage of component fuzzy sets
 198 in all dimensions in space.

199 Another method for combining two fuzzy member-
 200 ship functions was proposed by Mahardika P. as fol-
 201 lows¹⁸:

$$C_+ = \frac{N_1 C_1 + N_2 C_2}{N_1 + N_2} \quad (7)$$

$$S = \left(\frac{N_1 S_1^{-1} + N_2 S_2^{-1}}{N_1 + N_2} \right)^{-1} \quad (8)$$

202 where C_1, C_2, S_1, S_2 are the coordinates of the points
 203 of displacement of the centers and the matrices of the
 204 coefficients of the latitudes of the component mem-
 205 bership functions. C_+, S_+ - coordinates of the cen-
 206 ter offset point and matrix of latitude coefficients. $N_1,$
 207 N_2 are the corresponding proportional weights of the
 208 component fuzzy sets.

209 **Proposed Method to combine two member-**
 210 **ship functions**

211 The goal of combining two fuzzy sets is to replace two
 212 multivariable Gaussian fuzzy sets with a new multi-
 213 variable Gaussian fuzzy set. The composite fuzzy set
 214 must cover two composite fuzzy sets on the α -section.
 215 Since the multivariate Gaussian membership function
 216 is built on the basis of the multivariate Gaussian dis-
 217 tribution, we construct a method for combining two
 218 multivariate membership functions based on the syn-
 219 thesis of two multivariate Gaussian normal distribu-
 220 tions.

The center and covariance matrix representing the
 distribution of a set of N data points X_1, X_2, \dots, X_N
 in n -dimensional space is determined by the follow-
 ing formula:

$$C_N = \frac{1}{N} \sum_{i=1}^N X_i \quad (9)$$

$$\Sigma_N = \frac{1}{N} \sum_{i=1}^N (X_i - C_N)^T (X_i - C_N) \quad (10)$$

where X_i is of size $(1 \times n)$, is the i -th data point in
 dataset N . C_N is of size $(1 \times n)$, which is the center
 of the multivariate Gaussian distribution. Σ_N of size
 $(n \times n)$ is the covariance matrix of the multivariate
 Gaussian distribution.

Suppose we have two multivariate distributions $G_1 =$
 $\{C_{N_1}, \Sigma_{N_1}\}$ and $G_2 = \{C_{N_2}, \Sigma_{N_2}\}$ representing the dis-
 tributions of two sets of N_1 and N_2 distinct data, re-
 spectively.

$$C_{N_1} = \frac{1}{N_1} \sum_{i=1}^{N_1} X_i \quad (11)$$

$$\Sigma_{N_1} = \frac{1}{N_1} \sum_{i=1}^{N_1} (X_i - C_{N_1})^T (X_i - C_{N_1}) \quad (12)$$

$$C_{N_2} = \frac{1}{N_2} \sum_{i=1}^{N_2} Y_i \quad (13)$$

$$\Sigma_{N_2} = \frac{1}{N_2} \sum_{i=1}^{N_2} (Y_i - C_{N_2})^T (Y_i - C_{N_2}) \quad (14)$$

The union of G_1 and G_2 is understood as finding the
 multivariate distribution of G_+ , which represents the
 $N_1 + N_2$ distribution of their data points. The center
 and covariance matrix G_+ are defined as follows:

$$C_+ = \frac{\sum_{i=1}^{N_1} X_i + \sum_{i=1}^{N_2} Y_i}{N_1 + N_2} \quad (15)$$

$$= \frac{N_1 C_{N_1} + N_2 C_{N_2}}{N_1 + N_2}$$

$$\Sigma_+ = \frac{\sum_{i=1}^{N_1} (X_i + C_+)^T (X_i - C_+)}{N_1 + N_2} \quad (16)$$

$$+ \frac{\sum_{i=1}^{N_2} (Y_i + C_+)^T (Y_i - C_+)}{N_1 + N_2}$$

We replace: $X_i - C_+ = (X_i - C_{N_1}) + (C_{N_1} - C_+)$, we
 get:

$$\begin{aligned} & \sum_{i=1}^{N_1} (X_i - C_+)^T (X_i - C_+) \\ &= \sum_{i=1}^{N_1} [(X_i - C_{N_1}) + (C_{N_1} - C_+)]^T \times \\ & [(X_i - C_{N_1}) + (C_{N_1} - C_+)] \\ &= \sum_{i=1}^{N_1} [(X_i - C_{N_1}) + (X_i - C_{N_1})]^T + \\ & \sum_{i=1}^{N_1} (X_i - C_{N_1})^T (C_{N_1} - C_+) + \\ & \sum_{i=1}^{N_1} (C_{N_1} - C_+)^T (X_i - C_{N_1}) + \\ & \sum_{i=1}^{N_1} (C_{N_1} - C_+)^T (C_{N_1} - C_+) \\ &= \sum_{i=1}^{N_1} (X_i - C_{N_1})^T (X_{N_1} - C_{N_1}) + \\ & \left(\sum_{i=1}^{N_1} (X_i - C_{N_1})^T \right) (C_{N_1} - C_+) + \\ & (C_{N_1} - C_+)^T \sum_{i=1}^{N_1} (X_i - C_{N_1}) + \\ & N_1 * (C_{N_1} - C_+)^T (C_{N_1} - C_+) \end{aligned} \quad (17)$$

240 Since C_{N_1} is the center of G_1 , then

$$\sum_{i=1}^{N_1} (X_i - C_{N_1})^T = 0 \quad (18)$$

241 And

$$\sum_{i=1}^{N_1} (X_i - C_{N_1}) = 0 \quad (19)$$

242 Substituting (12), (18) and (19) into (17), we get:

$$\begin{aligned} & \sum_{i=1}^{N_1} (X_i - C_+)^T (X_i - C_+) \\ &= N_1 * \Sigma_{N_1} * (C_{N_1} - C_+)^T (C_{N_1} - C_+) \end{aligned} \quad (20)$$

243 Similar analysis

$$\begin{aligned} & \sum_{i=1}^{N_2} (Y_i - C_+)^T (Y_i - C_+) \\ &= N_2 * \Sigma_{N_2} * (C_{N_2} - C_+)^T (C_{N_2} - C_+) \end{aligned} \quad (21)$$

244 Substituting (20) and (21) into (16), we get:

$$\begin{aligned} \Sigma_+ &= \frac{N_1 * \Sigma_{N_1} + N_2 * \Sigma_{N_2} +}{N_1 + N_2} \\ & \frac{N_1 * (C_{N_1} - C_+)^T (C_{N_1} - C_+) +}{N_1 + N_2} \\ & \frac{N_2 * (C_{N_2} - C_+)^T (C_{N_2} - C_+)}{N_1 + N_2} \end{aligned} \quad (22)$$

245 It is easy to see that the center G_+ is a point on the
246 line connecting the centers G_1 and G_2 , and divides
247 this segment into two segments corresponding to:

$$C_{N_1} - C_+ = \frac{N_2}{N_1 + N_2} (C_{N_1} - C_{N_2}) \quad (23)$$

$$C_{N_2} - C_+ = \frac{N_1}{N_1 + N_2} (C_{N_1} - C_{N_2}) \quad (24)$$

248 Substituting (23) and (24) into (22), we get:

$$\begin{aligned} \Sigma_+ &= \frac{N_1 * \Sigma_{N_1} + N_2 * \Sigma_{N_2} +}{N_1 + N_2} \\ & \frac{N_1 N_2}{(N_1 + N_2)^2} (C_{N_1} - C_{N_2})^T (C_{N_1} - C_{N_2}) \end{aligned} \quad (25)$$

249 Formula (25) gives us the covariance matrix of the
250 multivariate distribution G_+ through the center and
251 the covariance matrix of the original distributions.

252 Unlike combining two distributions with multiple
253 variables, when combining two fuzzy sets, the cover-
254 age aspect of the combined fuzzy set must also be
255 taken into account.

256 Apply a formula similar to (25) to combine two fuzzy
257 sets with centers C_1, C_2 and the corresponding ma-
258 trix of expansion coefficients S_1, S_2 . Values N_1, N_2
259 are replaced by $\det(S_1), \det(S_2)$:

$$\begin{aligned} S_+ &= \frac{\det(S_1) * S_1 + \det(S_2) * S_2}{\det(S_1) + \det(S_2)} \\ &+ \frac{\det(S_1) * \det(S_2)}{(\det(S_1) + \det(S_2))^2} (C_1 - C_2)^T (C_1 - C_2) \end{aligned} \quad (26)$$

260 The center of the fuzzy set of the sum is transformed
261 from formula (15) as follows¹³:

$$C_+ = \frac{\det(S_1) * C_1 + \det(S_2) * C_2}{\det(S_1) + \det(S_2)} \quad (27)$$

262 As suggested above, we use a sufficiently small α -
263 slice to define a cover of a multidimensional Gaussian
264 fuzzy set. The criterion for the combined fuzzy set is
265 that the cut area α must be a covering of two com-
266 posite fuzzy sets. The results of combining two fuzzy
267 membership functions according to the formula (26)
268 are shown in Figure 5.

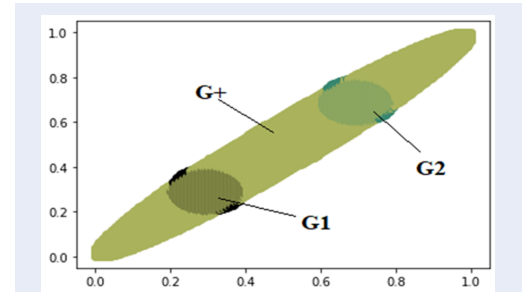


Figure 5: Multivariate Gaussian aggregation by formulas (26)

269 It is easy to see that the coverage of the combined fuzzy
270 set does not correspond to the coverage of the two
271 original fuzzy sets. To analyze the influence of the
272 components of formula (22) on the coverage of the
273 combined fuzzy set, we proceed to the union of two
274 fuzzy sets with its first term (equivalent to Mahardik
275 P's algorithm):

$$S_+ = \frac{\det(S_1) * S_1 + \det(S_2) * S_2}{\det(S_1) + \det(S_2)}$$

276 The results of combining two fuzzy sets using the first
277 term of formula (26) are shown in Figure 6.

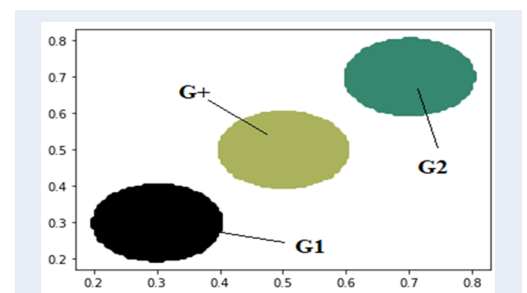


Figure 6: Multivariate Gaussian aggregation by the first term of formula (26)

278 Accordingly, we see that the first term allows us to
279 average the coverage of component distributions, and

280 the second element allows us to expand the coverage
 281 along the line connecting their centers.
 282 To generate the most appropriate aggregate function,
 283 we will use the following union formula:

$$D = \frac{\overrightarrow{C_1 C_2}}{2}$$

$$C_+ = C_1 + D + \left(\overrightarrow{r_\alpha^1} - \overrightarrow{r_\alpha^2} \right) / 2 \quad (28)$$

$$S_+ = \frac{\det(S_1) * S_1 + \det(S_2) * S_2}{\det(S_1) + \det(S_2)} - \frac{D^T D}{2 * \ln(\alpha)} \quad (29)$$

284 where C_+ , C_1 , C_2 are the centers of the new fuzzy set
 285 and two component fuzzy sets, respectively, $\overrightarrow{r_\alpha^1}$ is the
 286 radius vector of the first partial fuzzy set on the section
 287 α in the direction $\overrightarrow{C_1 C_2}$, $\overrightarrow{r_\alpha^2}$ is the radius vector
 288 of the second component fuzzy set on the section
 289 α in the direction $\overrightarrow{C_2 C_1}$. All of the above quantities
 290 have size $(1 \times n)$, where n is the number of dimensions
 291 of the input space. S_+ , S_1 , S_2 of size $(n \times n)$ is the
 292 covariance matrix of the new multidimensional fuzzy
 293 set and two-component fuzzy set, respectively, D is
 294 the radius vector in the direction $\overrightarrow{C_1 C_2}$ on the section
 295 α of the total fuzzy set. The $D^T D$ multiplication is the
 296 $(n \times 1) * (1 \times n)$ matrix multiplication.

297 The idea of the union method is to determine the ex-
 298 treme point of two fuzzy rules in the direction $\overrightarrow{C_2 C_1}$,
 299 the center of the entire fuzzy set is the midpoint of the
 300 above two points, and the radius is a vector from the
 301 center to one of the two points.

302 Using the projection of the multivariable Gaussian
 303 function onto a plane perpendicular to the input plane
 304 and passing through the centers of the two combined
 305 fuzzy sets, we obtain the following one-dimensional
 306 Gaussian function:

$$H_1(x) = e^{-0.5 \frac{(x-C)^2}{\sigma^2}}$$

307 The results of combining two fuzzy membership func-
 308 tions according to the formula (29) are shown in Fig-
 309 ure 7.

310 We need to determine the value of σ so that the new
 311 membership function has α at the point x_0 (stretch
 312 the membership function to position x_0):

$$\begin{aligned} H(x_0) = \alpha &\Leftrightarrow e^{-0.5 \frac{x_0^2}{\sigma^2}} = \alpha \\ \Leftrightarrow \sigma^2 &= \frac{x_0^2}{-2 \ln(\alpha)} \text{ with } (0 < \alpha < 1) \end{aligned} \quad (30)$$

313 Based on this, we expand and experiment to construct
 314 formula (29).

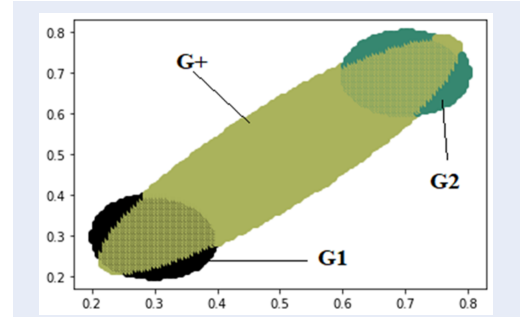


Figure 7: The union of two multivariable Gaussian functions by formula (29)

RESULTS

To evaluate the performance of the proposed method for combining fuzzy membership functions, a comparison is made with other methods. The initial conditions for modeling algorithms are the same:

$n_term = 10$; $loss_threshold = 0.01$; $loss_max = 1$

The fuzzy rule has the form

$$R^i : \text{If } x \in X^i(C_i, S_i) \text{ then } y_i = c_i^0$$

The training data is a set of 25 values taken at regular intervals in the interval $[0; 4] \times [0; 4]$. The algorithm stops when 10 fuzzy rules are reached.

Table 1 displays the comparison results of combining two membership functions using the proposed method and existing methods.

$$O_{Rule} = \sum_{i=1}^m |\hat{y}(C_i) - y(C_i)|$$

where O_{Rule} is an error according to fuzzy rules, C_i is the coordinates of the center of fuzzy rule i , $y(C_i)$ is the model output at C_i , $y(C_i)$ is the true value of the linear function at C_i , input, m - is the number of fuzzy rules.

$$O_{Model} = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

where O_{Model} is the model error during testing, \hat{Y}_i is the model output at the i -th observation, Y_i is the true value of the linear function of the i -th observation, n is the number of observations.

DISCUSSION

Shrinath G.A. method has a much larger error than the other two methods. It is explained that this method was created for building fuzzy neural models without taking into account the overlap between fuzzy rules and is applied to models in which the input data are independent. This union method creates

Table 1: Results of the application of methods to combine two fuzzy membership functions

Mode type	Criteria	Method Shrinath G. A.	Method Mahardik	Suggested method
Linear model	OModel	4.343	1.466	0.651
	ORule	642.330	0.820	0.328
Square model	OModel	33.68	0.715	0.255
	ORule	146.34	0.974	0.958

a union fuzzy set that spans the component fuzzy sets but does not take into account the relative positions of the component fuzzy sets. This results in a combined fuzzy set containing too many "redundancies", which increases the intersection between fuzzy rules.

Mahardik P's pooling method takes the average of fuzzy sets of components. The advantages of this method are the simplicity of calculation and the restriction of superposition between fuzzy sets, but this method has the following disadvantages:

1. Many points in the input space are in two-component fuzzy sets, but not in the combined fuzzy set, which leads to an incomplete description of the information.

2. In the case when the extended matrices of membership functions belonging to a partial fuzzy set are diagonal matrices, the extended matrix of the combined fuzzy set is also a diagonal matrix. Therefore, the combined fuzzy set cannot represent the relative positions of the fuzzy sets of components.

The proposed union method is an improved version of Mahardika's method. This method extends the result of the Mahardika method along a line connecting two centers of a partial fuzzy set until they coincide with their extreme points. Thus, the proposed method can show the relative position between two component fuzzy sets without increasing the overlap between fuzzy rules.

The simulation results show that the proposed method of combining fuzzy sets improves the accuracy of the neuro-fuzzy model and the independent operation of the generated fuzzy rules compared to the Mahardika method by increasing the computational complexity.

CONCLUSION

In the content of the article, we have analyzed the advantages and disadvantages of the multi-dimensional association functions used up to now, in addition, we have analyzed the advantages and disadvantages of the methods of combining the membership multi-variable functions. On the theoretical basis, we have

developed a new method of aggregating multivariable membership functions. The use of this method of combining multivariable membership functions in the neuro-fuzzy model synthesis algorithm allows reducing the average error of fuzzy rules during independent work from 0.82 to 0.328 and reducing the total error of fuzzy rules of the model from 1.466 to 0.651 when solving modeling problems compared to the Mahardika method. The content of the article creates a basis for applying multidimensional membership functions to the problem of building a fuzzy neural system in practice. In the future, we will aim to develop a theoretical system to be able to apply multidimensional membership functions to describe complex objects, to create more intelligent systems.

CONFLICT OF INTEREST

The authors assure that there is no conflict of interest in publishing the article.

AUTHORS' CONTRIBUTION

Bui Truong An participated in coming up with ideas for writing articles, collecting data and writing the manuscript.

Pham Thi Nguyen contributed to data interpretation and proofreading the article.

Pham Tuan Anh contributed to the Vietnamese - English translation and edited the article format.

REFERENCES

- Dutta A, Nayak A, Aditya, Panda RR, Nagwani NK. A Neuro Fuzzy System Based Inflation Prediction of Agricultural Commodities. In: 11th ICCNT. Electronic ISBN: 978-1-7281-6851-7. October 2020; Available from: <https://doi.org/10.1109/ICCNT49239.2020.9225453>.
- Ivanova D, Dejanov M. Fuzzy Logic Control Design Based on the Genetic Algorithm for a Modular Servo System. In: 17th Conference on Electrical Machines, Drives and Power Systems (ELMA). Electronic ISBN: 978-1-6654-3582-6. August 2021; Available from: <https://doi.org/10.1109/ELMA52514.2021.9503052>.
- Kien CV, Son NN, Anh HPH. Identification of 2-DOF pneumatic artificial muscle system with multilayer fuzzy logic and differential evolution algorithm. In: 12th IEEE Conference on Industrial Electronics and Applications (ICIEA). Electronic ISBN: 978-1-5090-6161-7. February 2018;

- 427 4. Sharma S, Kalra U, Srivathsan S, Rana KPS, Kumar V. Efficient air
428 pollutants prediction using ANFIS trained by Modified PSO al-
429 gorithm. In: 4th International Conference on Reliability, In-
430 form Technologies and Optimization (ICRITO). Electronic ISBN:
431 978-1-4673-7231-2. December 2015; Available from: [https://](https://doi.org/10.1109/ICRITO.2015.7359316)
432 doi.org/10.1109/ICRITO.2015.7359316.
- 433 5. Juang CF, Lin CH, Bui TB. Multiobjective Rule-Based Cooper-
434 ative Continuous Ant Colony Optimized Fuzzy Systems With a
435 Robot Control Application. IEEE Transactions on Cybernetics.
436 2018 Oct;50:650-663; PMID: 30296249. Available from: [https://](https://doi.org/10.1109/TCYB.2018.2870981)
437 doi.org/10.1109/TCYB.2018.2870981.
- 438 6. Chou KP, Lin CT, Lin WC. A self-adaptive artificial bee colony
439 algorithm with local search for TSK-type neuro-fuzzy system
440 training. In: 2019 IEEE Congress on Evolutionary Computation
441 (CEC). Electronic ISBN: 978-1-7281-2153-6. August 2019; Avail-
442 able from: <https://doi.org/10.1109/CEC.2019.8790334>.
- 443 7. Jin Q, Wang C, Wang H, Cai W, Niu Y. Hybrid Fuzzy Cuckoo
444 Search Algorithm for MIMO Hammerstein Model Identifica-
445 tion Under Heavy-Tailed Noises. In: 37th Chinese Control
446 Conference (CCC). Electronic ISBN: 978-988-15639-5-8. Octo-
447 ber 2018; PMID: 30043400. Available from: [https://doi.org/10.](https://doi.org/10.23919/ChiCC.2018.8482855)
448 [23919/ChiCC.2018.8482855](https://doi.org/10.23919/ChiCC.2018.8482855).
- 449 8. Nurcahyono AH, Nhita F, Saepudin D, Aditsania A. Price Pre-
450 diction of Chili in Bandung Regency Using Support Vector
451 Machine (SVM) Optimized with an Adaptive Neuro-Fuzzy In-
452 ference System (ANFIS). In: 7th International Conference on
453 Information and Communication Technology (ICoICT). Elec-
454 tronic ISBN: 978-1-5386-8052-0. September 2019; Available
455 from: <https://doi.org/10.1109/ICoICT.2019.8835367>.
- 456 9. Abdusamad A, Aburakhis M. Adaptive Control of Nonlinear
457 Systems Represented by Extreme Learning Machine (ELM)
458 and the Fuzzy Logic Control (FLC). In: IEEE 1st International
459 Maghreb Meeting of the Conference on Sciences and Techni-
460 ques of Automatic Control and Computer Engineering MI-
461 STA. Electronic ISBN: 978-1-6654-1856-0. June 2021; Available
462 from: [https://doi.org/10.1109/MI-](https://doi.org/10.1109/MI-STA52233.2021.9464386)
463 [STA52233.2021.9464386](https://doi.org/10.1109/MI-STA52233.2021.9464386).
- 464 10. Abonyi J, Babuska R, Szeifert F. Fuzzy modeling with multi-
465 variate membership functions: gray-box identification and
466 control design. IEEE Transactions on Systems, Man, and Cy-
467 bernetics, Part B (Cybernetics). 2001 Oct;31:755-767; PMID:
468 18244840. Available from: [https://doi.org/10.1109/3477.](https://doi.org/10.1109/3477.956037)
469 [956037](https://doi.org/10.1109/3477.956037).
- 470 11. Kang D, Yoo W, Won S. Multivariable TS fuzzy model identifi-
471 cation based on mixture of Gaussians. In: 2007 International
472 Conference on Control, Automation and Systems. December
473 2007;.
- 474 12. Lemos A, Caminhas W, Gomide F. Multivariable Gaussian
475 Evolving Fuzzy Modeling System. IEEE Transactions on Fuzzy
476 Systems. 2010 Oct;19:91-104; Available from: [https://doi.org/](https://doi.org/10.1109/TFUZZ.2010.2087381)
477 [10.1109/TFUZZ.2010.2087381](https://doi.org/10.1109/TFUZZ.2010.2087381).
- 478 13. Pratama M, Anavatti SG, Angelov PP, Lughofer E. PANFIS:
479 A Novel Incremental Learning Machine. IEEE Transactions
480 on Neural Networks and Learning Systems. 2013 Jul;25:55-
481 68; PMID: 24806644. Available from: [https://doi.org/10.1109/](https://doi.org/10.1109/TNNLS.2013.2271933)
482 [TNNLS.2013.2271933](https://doi.org/10.1109/TNNLS.2013.2271933).
- 483 14. Basil JMA, Al-Hadithi M, Jiménez A. Multidimensional mem-
484 bership functions in T-S fuzzy models for modelling and
485 identification of nonlinear multivariable systems using ge-
486 netic algorithms. Applied Soft Computing. 2019 Feb;75:607-
487 615; Available from: <https://doi.org/10.1016/j.asoc.2018.11.034>.
- 488 15. Abonyi J, Babuska R, Szeifert F. Fuzzy modeling with multi-
489 variate membership functions: gray-box identification and
490 control design. IEEE Transactions on Systems, Man, and Cy-
491 bernetics, Part B (Cybernetics). 2001 Oct;31:755-767; PMID:
492 18244840. Available from: [https://doi.org/10.1109/3477.](https://doi.org/10.1109/3477.956037)
493 [956037](https://doi.org/10.1109/3477.956037).
- 494 16. Mahalanobis PC. On the generalised distance in statistics.
495 Proceedings of the National Institute of Sciences of India.
496 1936;2(1):49-55;.
- 497 17. Ferdaus MM, Pratama M, Anavatti SG, Garratt MA, Pan Y.
Generic Evolving Self-Organizing Neuro-Fuzzy Control of
Bio-Inspired Unmanned Aerial Vehicles. IEEE Transactions
on Fuzzy Systems. 2020 Aug;28(8):1542-1556; Available from:
<https://doi.org/10.1109/TFUZZ.2019.2917808>.
18. Pratama M, Anavatti SG, Lughofer E. GENEFIS: Toward an
Effective Localist Network. IEEE Transactions on Fuzzy Sys-
tems. 2014 Jun;22(3):547-562; Available from: [https://doi.org/](https://doi.org/10.1109/TFUZZ.2013.2264938)
[10.1109/TFUZZ.2013.2264938](https://doi.org/10.1109/TFUZZ.2013.2264938).

Ứng dụng của hàm liên thuộc đa biến vào mô hình nơ-ron mờ

Bùi Trường An^{1,*}, Phạm Tuấn Anh¹, Phạm Thị Nguyên²



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TÓM TẮT

Trong những năm gần đây, các hệ thần kinh mờ ngày càng trở nên phổ biến nhờ khả năng học tập và diễn giải mạnh mẽ của chúng. Trong lĩnh vực điều khiển, hệ thần kinh mờ tỏ ra vượt trội hơn các hệ thống thông minh khác. Ngoài ra, lý thuyết về sự kết hợp giữa mạng nơ-ron và logic mờ còn được sử dụng trong nhiều lĩnh vực khác như dự đoán, mô phỏng, hỗ trợ quyết định, v.v. Tuy nhiên, hầu hết các hệ thống nơ-ron mờ được sử dụng ngày nay đều là sự kết hợp giữa mạng nơ-ron và các hàm liên thuộc mờ đơn biến. Các hàm này có ưu điểm là đơn giản, dễ thiết lập nhưng đi kèm với đó là thiếu khả năng diễn giải đối với các đối tượng phức tạp. Đối với các đối tượng cần được mô tả bằng hai đại lượng trở lên, các hàm liên thuộc đơn chiều không thể biểu diễn nó. Việc áp dụng hàm thành viên đa biến là cần thiết trong trường hợp này. Việc ứng dụng hàm thành viên nhiều biến gặp nhiều rào cản do tính phức tạp, các thuật toán áp dụng hàm thành viên nhiều biến còn sơ sài và chưa phát huy hết ưu điểm của chúng. Trong bài viết này, chúng tôi sẽ giới thiệu một phương pháp áp dụng hàm thành viên Gaussian đa biến cho phép cải thiện hiệu suất mô phỏng so với các phương pháp đã giới thiệu trước đó.

Từ khoá: Tập mờ, hàm liên thuộc đa biến, hàm Gaussian, mô hình thần kinh mờ

¹Viện Công nghệ thông tin, Thành phố Hồ Chí Minh, Việt Nam.

²Trường Đại học Tài nguyên Môi trường thành phố Hồ Chí Minh, Thành phố Hồ Chí Minh, Việt Nam.

Liên hệ

Bùi Trường An, Viện Công nghệ thông tin, Thành phố Hồ Chí Minh, Việt Nam.

Email: buitruonganmta92@gmail.com

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